THE ESSENTIALS

OF

DESCRIPTIVE GEOMETRY

BY

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SECOND EDITION REVISED

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PREFACE

It has been the endeavor of the author in writing this text to include only those portions of descriptive geometry which possess industrial utility and which develop the qualities of mind so essential in a draftsman.

First and foremost descriptive geometry should aim to teach projection. A draftsman must be able both to read and to write drawings with facility, and projection is the very grammar of the language of the designer. Secondly, descriptive geometry should aim to develop the ability to solve problems concerning the relations of points, lines, and planes. These are, of course, but elementary parts of all engineering structures as shown on drawings and it is important that a draftsman be prepared to solve problems relating to them directly on the drawing board. Thirdly, and perhaps most important of all, descriptive geometry should aim to promote the ability to analyse a problem into its component parts, to reason from a given set of conditions to a required set of conclusions, and to build up from the drawing a mental picture of the object which is there represented, for without the ability to analyse, to reason, and to visualize a draftsman is lacking in the essential qualifications of his calling.

For these reasons the subject has been discussed from the point of view of a draftsman and the essential relations of points, lines, and planes have been treated in the third quadrant. The order of the material has been carefully considered and while there is some departure from traditional arrangement it is believed that the selection of material and its arrangement will be found both logical and conducive to a natural development of the subject. Many problems of a carefully graded character and of a practical "flavor" have been inserted at frequent intervals in the belief that such work is invaluable in fixing principles and promoting genuine interest.

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In the discussion on surfaces considerable variation may be found from other works on the subject both in content and treatment. It is believed, however, that the material included is broad enough in character for practical purposes; and it is hoped that the method of treatment, the character of the problems, and the discussion on model making will stimulate interest in this important and useful part of the subject.

In the preparation of this text the author has consulted and acknowledges his indebtedness to the following standard texts: Geometrie Descriptive by G. Monge; Theoretical and Practical Graphics by F. N. Willson; Elements of Descriptive Geometry by Albert E. Church; Elements of Descriptive Geometry by C. W. MacCord; Engineering Drawing by Thos. E. French.

                                                    F. G. HIGBEE.

Iowa City, Ia., *Feb.* 1, 1915.

PREFACE TO SECOND EDITION

THE second edition of this book presents no departure from the original endeavor of the author which was to discuss only the essentials of descriptive geometry.

The material in the first edition has been corrected and revised, some portions have been entirely rewritten, and at the request of a number of teachers who are using the book as a text a chapter on tangencies has been added. The changes from the first edition are those which practical classroom use has demonstrated as advisable.

The author wishes to acknowledge his indebtedness to his colleagues in the department of Descriptive Geometry and Drawing at the University of Iowa, to Dr. F. M. Comstock and his associates at the Case School of Applied Science, to Professor E. F. Chillman, Rensselaer Polytechnic Institute, and to many others who have aided in the preparation of this edition with helpful criticisms and suggestions.

                                            F. G. HIGBEE.

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CONTENTS

ESSENTIALS OF DESCRIPTIVE GEOMETRY

CHAPTER I

ORTHOGRAPHIC PROJECTION

1. Descriptive Geometry is the science of graphic representation. It is the means by which objects are shown on drawings and by which problems relating to these objects are solved.

2. When an object such as a machine, a bridge, a building, or any elemental part of such an engineering structure, is designed a drawing or set of drawings of it is made. Such a drawing is not only invaluable to the designer in recording his ideas step by step and in assisting him in his design but it is also indispensable to the artisan who constructs the object. A drawing, as used in the engineering sense, is a complete set of instructions from the designer to the workman by means of which the workman may reproduce, in exact shape and size and in material and finish, the identical object which the designer represented on the drawing.

The principles by which the shape and size of objects, and by which problems relating to such representations are graphically solved, are found in descriptive geometry. The art of putting on the drawing such additional information as dimensions, shop notes, finish, and other data regarding the construction of the object represented, is not a part of descriptive geometry but is included as a part of the art which is generally known as drafting.

3. There are two general systems by which graphic representations are made. One system has for its purpose the representation of objects as they appear, and is called *scenographic projection*. When an object is looked at from some particular

point of view one can get an idea of its shape and size because at
least three faces of the object are seen, and the effect of light and
shade serves to bring out the configuration, and because the eye
naturally compares and judges distances. If a representation
be made showing how the object appears to the observer, such a
drawing will be a *scenographic projection*, or, as it is more com-
monly called, a *perspective drawing*.

. There are also a number of other forms of projection which
aim at the same result, — that is, to represent the object as it
appears, — which may be called pseudo-perspective drawing.
Isometric, cabinet, and oblique projection are examples, but as
these are used as forms of perspective drawing and require
extensive treatment they will not be considered in this text.

4. The other system of projection has for its purpose not the
representation of objects as they appear, but the representation
of objects as they actually are. This form of representation is
called *orthographic projection;* and it is this form of projection
which is used in all engineering drawing where the purpose of the
drawing is to convey, from the draftsman to the workman, in-
formation and directions for building.

It is with orthographic projection and the use of orthographic
projection as a means of solving drawing-board problems graphi-
cally that this text has to do.

5. In perspective drawing the eye of the draftsman is assumed
to be at a definite distance from the object. Therefore the rays
of sight which travel from the eye to the corners and other parts
of the object converge to a point. If now a plane be set up be-
tween the eye and the object, as in Fig. 1, and the points where
the lines of sight pierce this plane be found and joined, the result
will be a picture of the object observed. It is obvious that the
picture will be smaller than the object itself, because the plane,
which is called the picture plane, is between the eye and the
object.

It will be observed, then, that in perspective drawing the size
of the picture will vary inversely as the distance of the picture
plane from the object; when it is between the object and the
eye the picture will be smaller than the object, and when it is on

the far side of the object the picture will be larger. This is due, of course, to the fact that the lines of sight converge to the eye.

It will be observed that the relation of the lines in the object to the corresponding lines in the picture is not the same for all
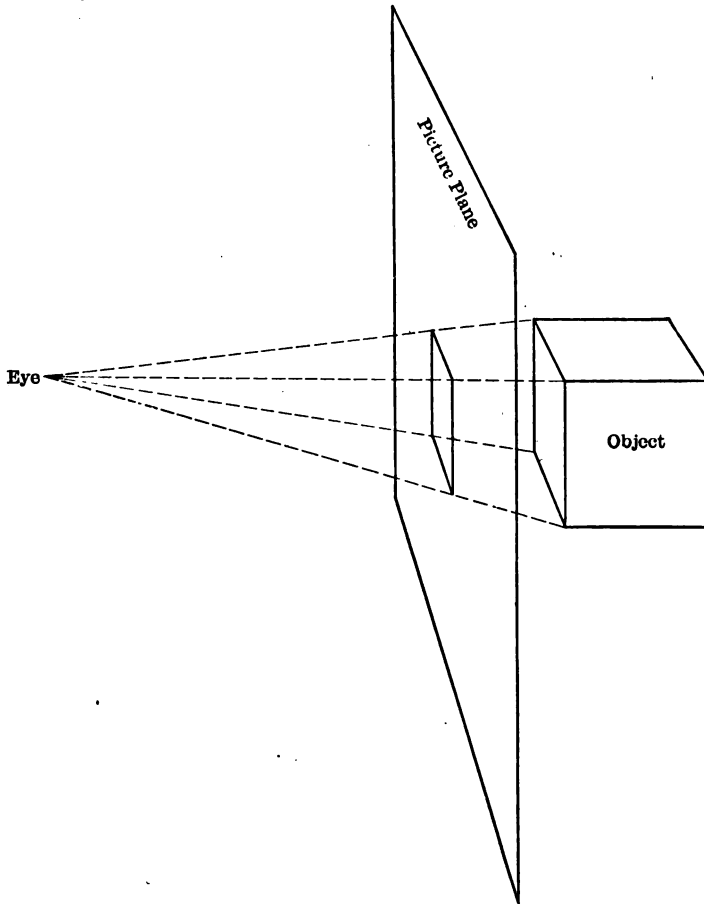
FIG. 1.

lines. In other words, some lines are foreshortened more than others owing to the position of the eye.

It will also be observed that in perspective more than one face of the object is seen from one point of view.

6. In engineering drawing — or in orthographic projection — the eye of the draftsman is assumed to be at infinity.   Therefore, the lines of sight do not converge but are parallel.   If now, as
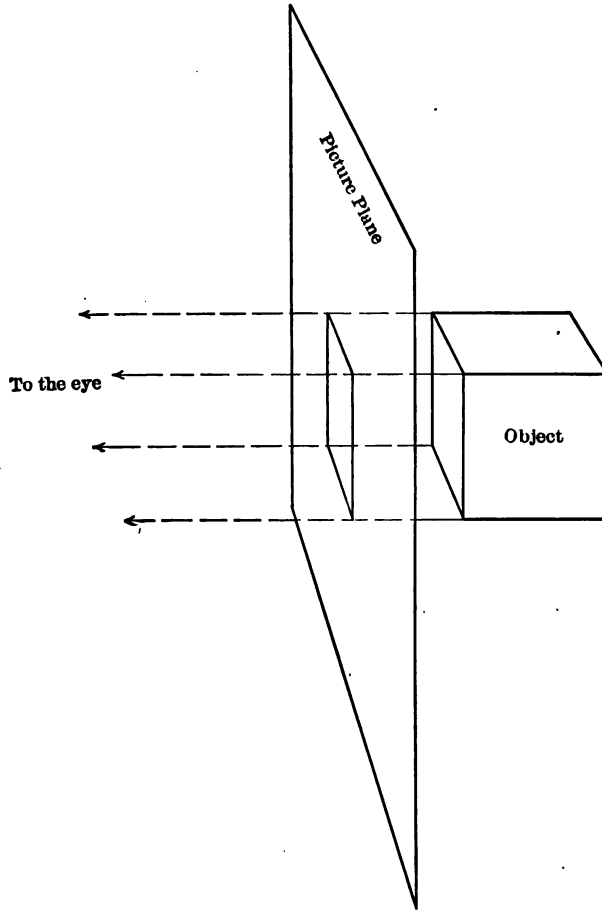
FIG. 2.

in Fig. 2, a plane be set up between the eye and the object and the points where the lines pierce this plane be found and joined the result will be a picture of the object not as it appears but as it actually is, and each line of the picture will be the same length as the corresponding line in the object.

It will be observed, therefore, that the size of the picture *does not vary* with the position of the picture plane.   No matter where the plane is placed the size of the resulting picture will be the same.   Of course, in actual drawing, objects are drawn to scale. That is, some part of an inch is taken on the drawing to represent an inch on the object in order that large objects may be drawn on convenient sized drawing sheets, but the reduction is the same for all lines.   If, for example, one line in the picture is drawn to a scale of $\frac{1}{4}'' = 1''$ all lines will be drawn to that same scale.

It will be observed, then, that the relation of lines in the object to the corresponding lines in the picture is the same for all lines and is the same no matter what distance the picture plane is from the object.   If the picture of a line be measured, therefore, it will be found to equal in length the line itself.

It will also be observed that in orthographic projection only one face of an object is visible from one point of view, and, therefore, that to show more than one face more than one picture must be drawn and more than one point of view assumed.

7. In orthographic projection the picture planes are called *planes of projection* and since more than one are required to show three dimensions of an object one has been assumed vertical and one has been assumed horizontal, because upon two planes an object may in general be projected so as to give a complete idea of its shape and size.   The horizontal plane is called the *horizontal plane of projection* and the view of the object obtained by . projecting it upon this plane is called the *plan,* or *the plan view,* or *the H projection.*   The vertical plane is called the *vertical plane of projection* and the view obtained by projecting the object upon this plane is called the *elevation, or the front view, or the V projection.*

8. Fig. 3 shows a picture of the two planes of projection with an object ready to project upon them.   The line where the H and V planes intersect is called the *ground line,* — hereafter denoted as G. L., — and in drawings the ground line is the line of reference which indicates the location of the object with reference to the planes of projection.

To project the object in Fig. 4 upon the H plane, or to get a

plan view of it, the eye of the draftsman is supposed to be so far above the H plane that the lines of sight are perpendicular to it.
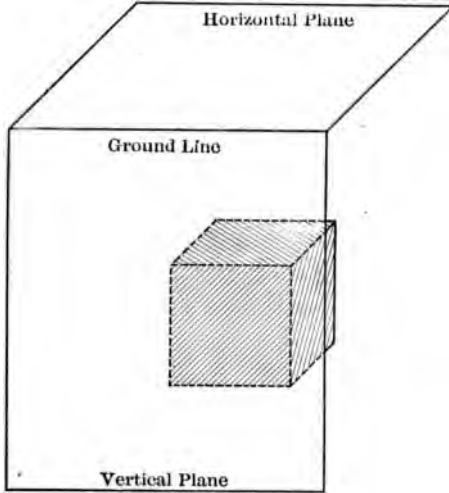
FIG. 3.

Therefore, draw perpendiculars from each corner of the object to the H plane and join the points where the perpendiculars pierce H. To project the object upon the V plane, or to get an elevation of it, the eye of the draftsman is assumed to be in front of the V plane and so far in front that the lines of sight are perpendicular to it. Therefore, draw perpendiculars from the object to the V plane and join the points where the perpendiculars pierce V. *It will be noted, therefore, that the projection of a point upon a plane is the foot of a perpendicular from the point to the plane.* It should be kept in mind, however, that Fig. 4 is merely a *picture*, not an actual representation, showing how projections are made.
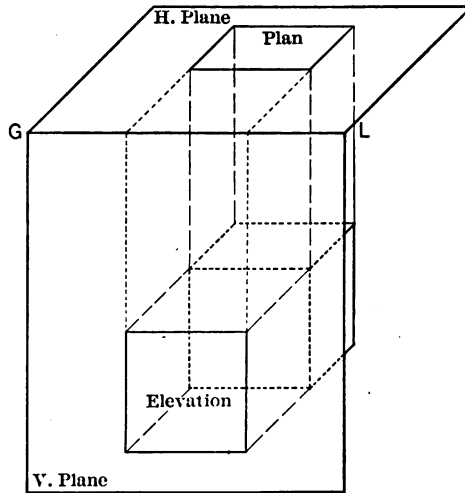
9. Now it will be observed that the two planes of projection stand at right angles to each other, and that if the projections or views which are shown on these planes are to be represented as they appear and shown on these planes are to be represented as they appear and

FIG. 4.

on one sheet of paper these two planes must be considered as coinciding with the paper on which the drawing is made. This is accomplished as shown in Fig. 5 by revolving H into coincidence with V. The H plane and the V plane are now coinciding with the paper, but it will be noted that the relation of the plan and the elevation with respect to each other and the

H. Plane

Plan

Plan

G L G L

Elevation
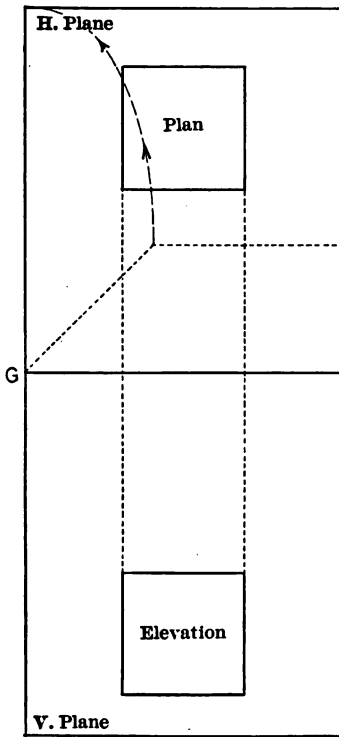
Elevation
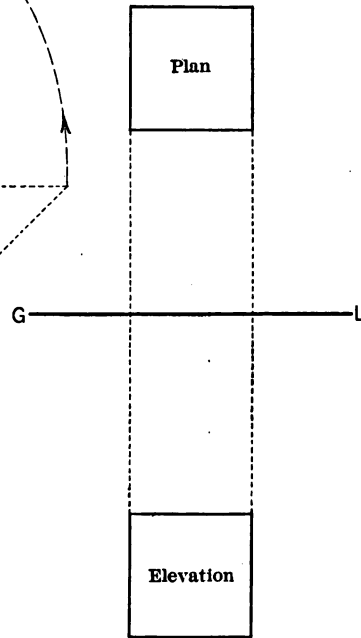
V. Plane

FIG. 5. FIG. 6.

ground line has in no way been changed by revolving the H plane; each remains the same distance from the ground line as before, and it will be noted carefully that the *plan and elevation of the same point lie in the same perpendicular to the ground line.*

In a drawing the planes of projection are not limited in extent, so no boundary lines need be shown for them. A ground line, as in Fig. 6, is drawn which indicates the position of the planes

with respect to the object, — it being understood that the H
plane is above the V plane, — and the plan and elevation of the
object are drawn in the required position by locating them at the
proper distances from the G. L.   Fig. 6 shows a plan and eleva-
tion of an object as usually drawn in descriptive geometry.

10. Now since the two planes of projection are indefinite in
extent, and stand at right angles to each other, it will be clear
that such a pair of
planes will divide
space into four quar-
ters as shown by the
pictorial drawing in
Fig. 7. These four
quarters are called
quadrants, or angles,
and for convenience
are designated as
First, Second, Third,
and Fourth Quad-
rants. Drawings may
be made with the ob-
ject in any of these
quadrants, but all
commercial drawing
is done either in the
third or first quad-
rant.   The reason for
this will be evident
from a study of **Fig.**
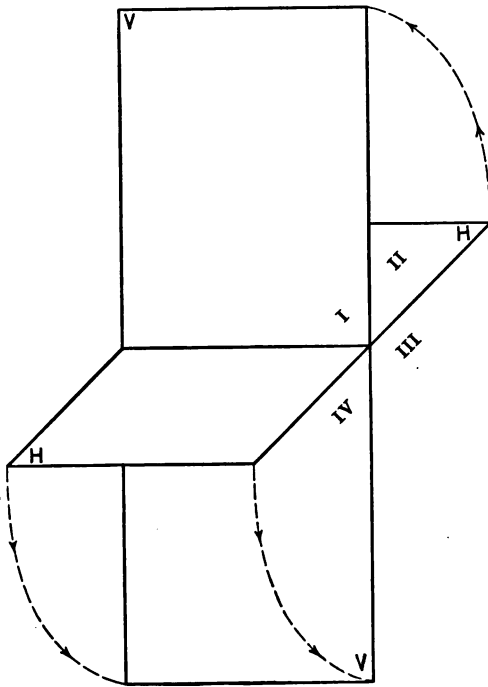
FIG. 7.

7A which shows how the quadrants are located with respect to
the G. L. when the planes of projection coincide with the paper.
From this figure it will be seen that in third quadrant drawing
the plan is above the elevation; in first quadrant drawing the
plan is below the elevation; and in second and fourth quadrant
drawing the plan and elevation are on the same side of the ground
line, and therefore often interfere with each other.

Numbering the quadrants is done to afford a convenient way

of indicating the location of an object with respect to the planes of projection. When an object is said to be in the first quadrant it means simply that it is above the H plane and in front of the V plane; therefore its plan view will be below the elevation. When an object is said to be in the third quadrant it means that

First Quadrant.   Second Quadrant.   Third Quadrant.   Fourth Quadrant.

Plan *below* G. L. and   H and V coinciding   Plan *above* G. L. and   H and V coinciding
elevation *above* G. L.   above G. L. There-   elevation below G. L.   below G. L. There-
   fore plan and eleva-   fore both views be-
   tion both *above* G. L.   low G. L.
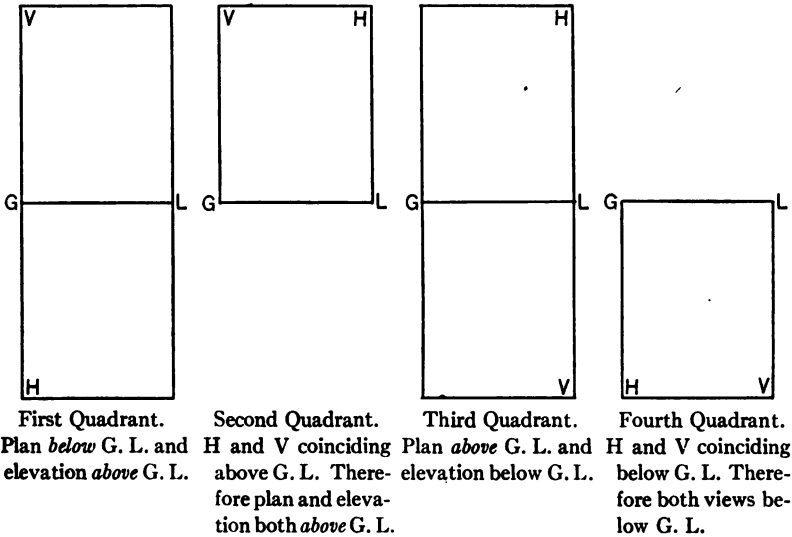
FIG. 7A.

it is below H and behind V; therefore its plan view will be above its elevation. Also when a drawing is examined these relations serve to assist in visualizing the object, and once the "language" of projection and drawing is learned these details are not thought of as such, and one reads the drawing much as one reads a sentence without analysing it into subject, object, verb, etc.

PROBLEMS IN PROJECTION

1. Draw the plan and elevation of a regular hexagonal prism whose base is 2″ in diameter and whose altitude is 3″.

2. Draw the plan and elevation of a regular pentagonal pyramid whose altitude is 3″ and the sides of whose base are 2″.

3. Draw the plan and elevation of a regular truncated hexagonal pyramid whose lower base is 3″ in diameter, whose upper base is 2″ in diameter,

and whose height is 2″.   Let the pyramid be in the first quadrant with its lower base on H.

4. Draw the plan and elevation of a 1″ by 2″ by 3″ block having three circular holes through it the 1″ way.   The holes are ½″ in diameter and are spaced with centers 1″ apart with the middle hole in the center of the 2″ by 1″ face of the block.

5. Draw the plan and elevation of a cube with 2″ edges.   Let the cube be in the first quadrant resting on H with its vertical faces inclining to V at angles of 30 and 60 degrees.

6. Draw the plan and elevation of a cast iron plate 9″ by 3′ by 6′.   In the center of the 3′ by 6′ face is a hole 6″ square and 4″ deep; in the 9″ by 6′ face are drilled two 3″ holes through the plate 8″ from each end and on the center line.

7. A grain hopper in the shape of a regular truncated square pyramid has an opening 4′ by 4′, and an outlet 12″ by 12″ which is 4′ below the opening.   Draw the plan and elevation of the hopper when the opening lies in the H plane with its edges inclining at angles of 45 degrees with the G. L.

8. Draw the plan and elevation of a plain box 4″ deep, 6″ wide and 9″ long with its lid half raised.   The box is built of ¾″ material and the lid is hinged along the 9″ edge.

9. A hood is fastened to a wall.   The opening of the hood is 4′ by 4′ and its outlet is into a square pipe 6″ in diameter also fastened to the wall directly above the opening.   The distance between the opening and outlet is 3′.   Draw the plan and elevation.

10. Draw the plan and elevation of an hexagonal pyramid whose base is 2″ in diameter and whose altitude is 3″.   The apex of the pyramid is directly above one corner of the base.

NOTE: Unless otherwise specified objects given in problems are to be drawn in the third quadrant; and when no distance from H and V is given the views may be located at any convenient distance from the ground line. It is well, however, to keep the distance between views relatively short so that the eye may readily note which points are projections of each other; in practical drafting only enough space is allowed between views to permit of placing dimensions and to keep the views distinct.

CHAPTER II

PROFILE PLANE

11. While in many cases a plan and elevation will show the shape and size of an object it quite frequently happens that a third view will be required for a complete representation. In such a case the H and V planes of projection are not sufficient and a third plane of projection must be added. The third view, which is most commonly used to supplement the plan and elevation, is called an *end view*, or a *side* or *end elevation*, or a *profile projection;* and the plane upon which this view is made is called the *profile plane*, or *P plane*.

12. In Fig. 8 is shown a picture of the arrangement of H, V, and P for the third quadrant. From this picture it will be clear that the profile
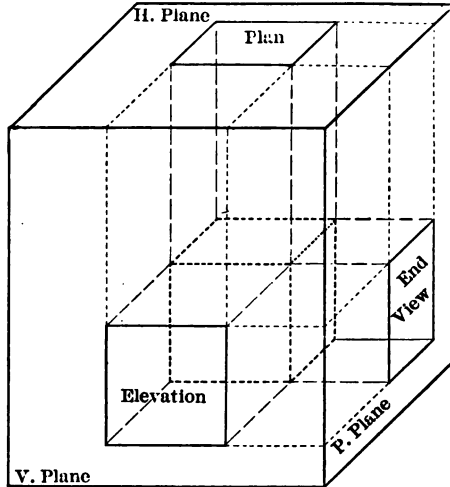
Fig. 8.

plane is perpendicular to the G. L., and therefore to both H and V, and it may be located at any convenient point on the G. L. and on either or both sides of the object. In case it is located on the right side of the object the view obtained will be the *right end view*. It not infrequently occurs in drafting that both end views are shown.

13. In Fig. 9 is shown how the H, V, and P planes, pictorially represented in Fig. 8, appear when they are revolved to

coincide with the plane of the drawing. From this figure it will be observed that the relations between H and V which have been previously discussed are in no way altered, that the profile plane takes a position to the right or left of the V plane, and that the
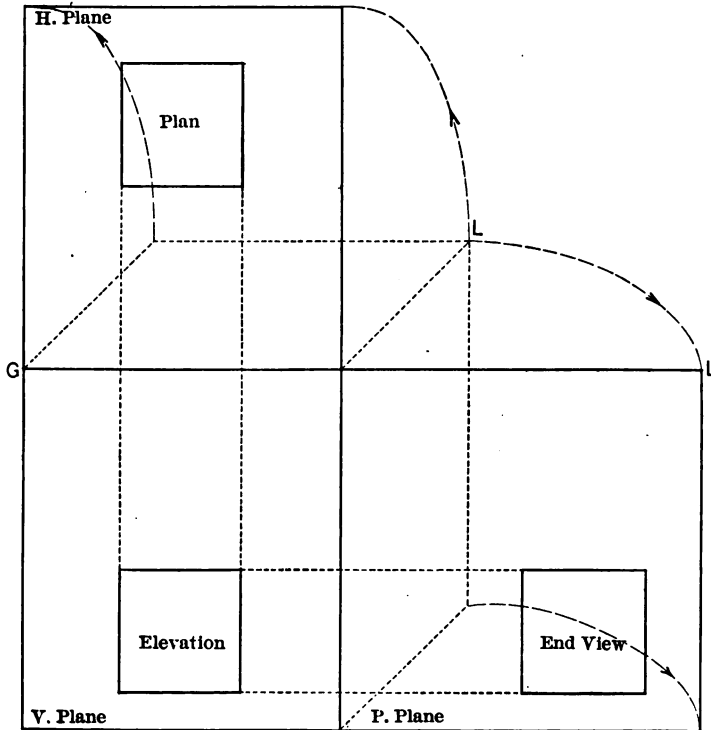
FIG. 9.

elevation and end view of the same point lie on lines which are parallel to the G. L.   It will also be observed that the end view shows how far an object is below H, by means of the distance the end view is below the G. L.; and also shows how far an object is behind V, by means of the distance the end view is to the right, or left, of the profile ground line.

There are, of course, two profile ground lines; one where the P plane cuts V, and the other where it cuts H.   In Fig. 9 it will be noted that the ground line between V and P is really an *end*

view of the V plane, and the one between H and P is really an *end view of H.* This latter coincides, after the planes are revolved, with the G. L. for H and V.

In actual work the three planes are not limited in extent and their location is determined only by the ground lines. Fig. 10
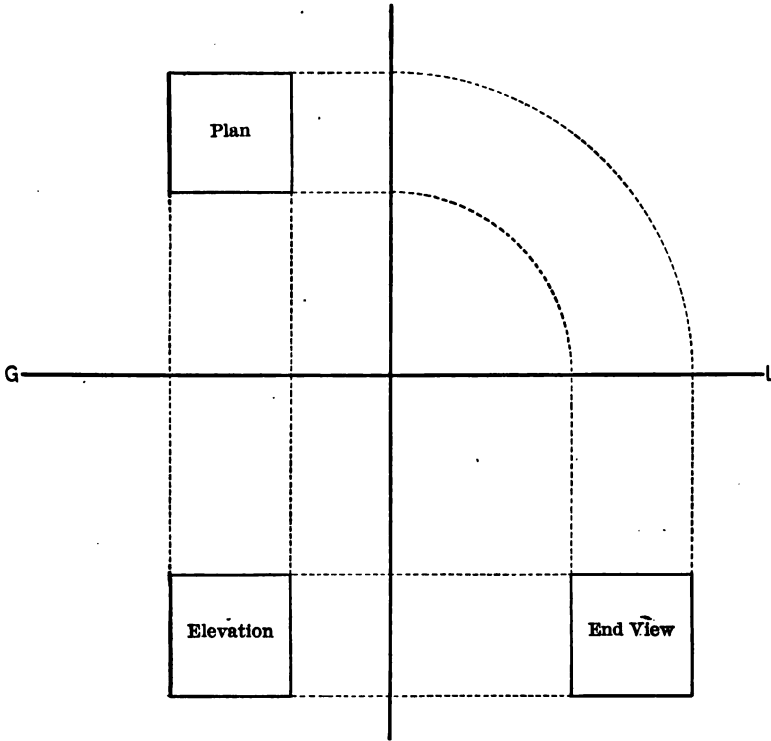
FIG. 10.

shows how Fig. 9 appears as usually drawn with only the ground lines showing the planes.

14. The arrangement of the three planes for all quadrants is shown pictorially in Fig. 11, with the profile plane on the right. Now it will be observed that if the profile plane be revolved so that the portion for the third quadrant fall to the right of the V plane the profile plane for the first quadrant will, of course, revolve in the opposite way, and will be coincident with V. This

is shown in Fig. 12 where a timber is partly in the first and partly in the third quadrant.

In practical drafting, however, the arrangement of the three planes for the first quadrant is as shown in Fig. 13 where the
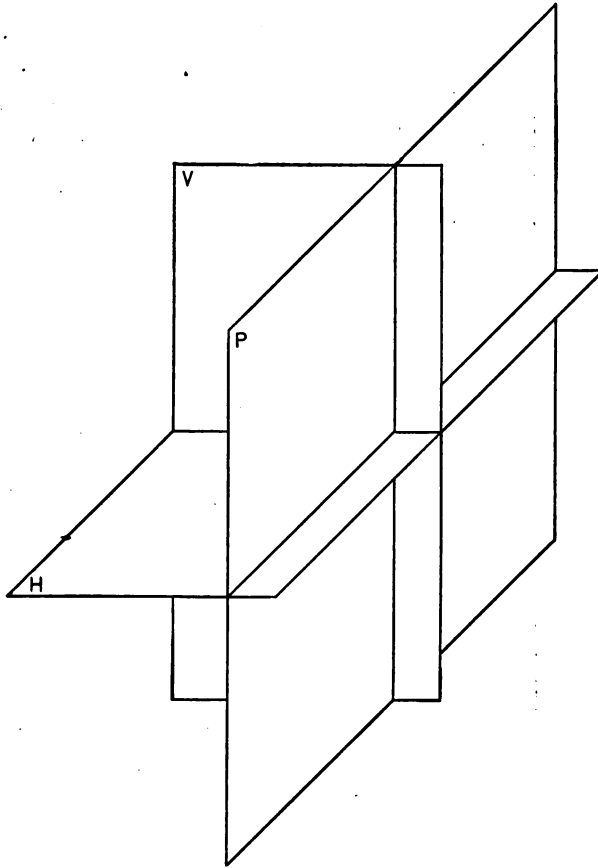
FIG. 11.

block shown at A is projected.   The P plane is revolved out of the way of the elevation in the same way as for the third quadrant, thus avoiding the confusion which might result from having the end view superimposed upon the elevation.

One of the problems constantly before the draftsman for solu-
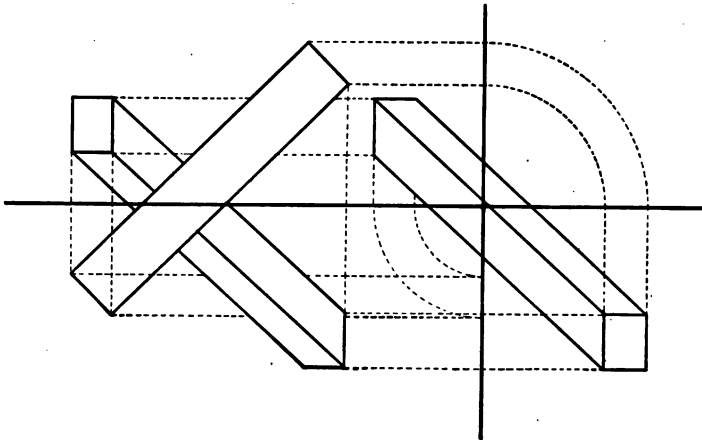
FIG. 12.
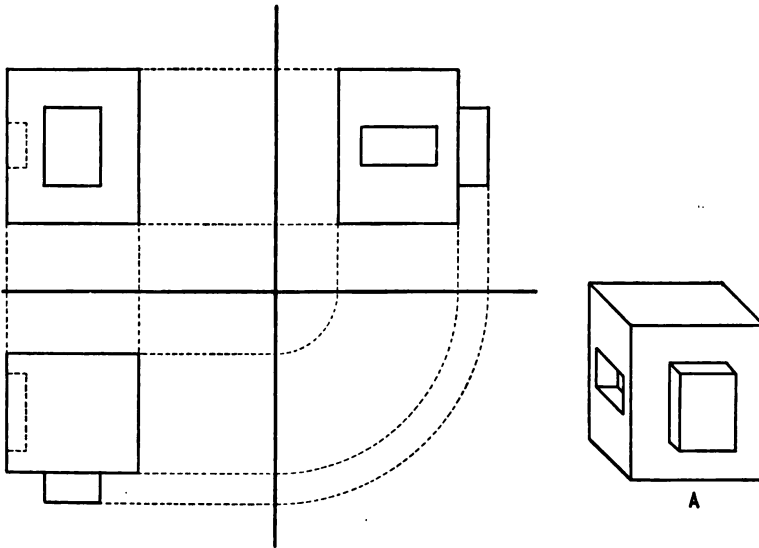
FIG. 13.

tion is the construction of the end views of an object which is shown only in plan and elevation. Fig. 14 shows the plan and elevation of an object of which it is desired to find both end views.

The profile ground lines are assumed at any convenient point along the given ground line as shown. Through the eleva-

tion of each point draw dotted lines parallel to the G. L.   On
these dotted lines, the proper distance from the profile ground
lines, will lie the end view of each point.   To find this location
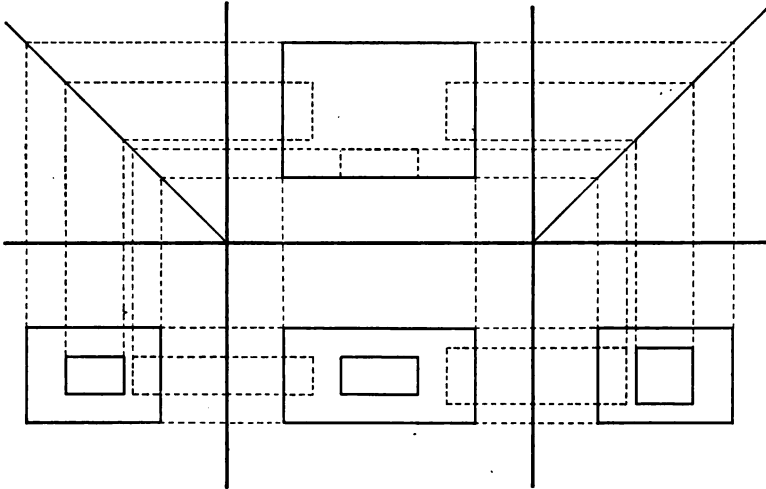
FIG. 14.
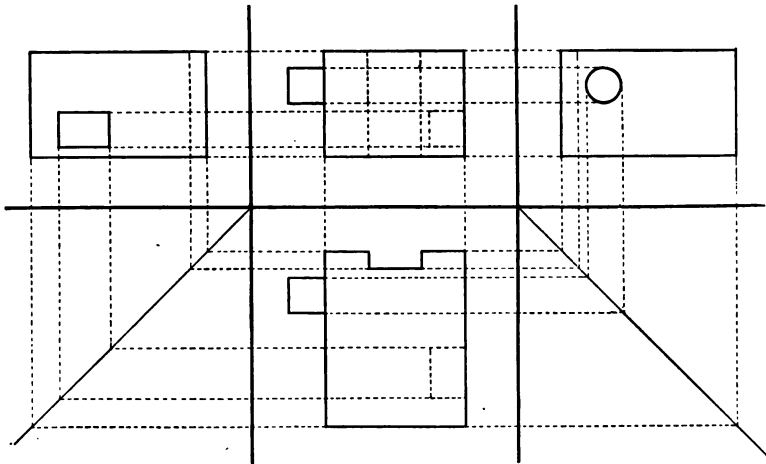
FIG. 15.

lay off on the dotted lines distances to the right and left of the
profile ground line in V, the distance each point is back of V.
This may be done conveniently by drawing a 45 degree line as
shown and drawing perpendiculars.   Since the distance each

end view is from the profile ground line in V is equal to the distance it is back of V, points so located will be the proper distance to the right or left of the profile G. L.

Fig. 15 shows a similar problem solved in the first quadrant and serves to show the arrangement of views for the first quadrant.

By using similar methods of projection a third view of any object may be obtained when two views are given. Thus: a plan view may be found from the elevation and end view; or an elevation may be found from the end view and plan.

PROBLEMS IN PROJECTION

11. Draw the plan, elevation, and right end view of a cube with edges 2″ long. Let the upper face of the cube be parallel to and 1″ below H with
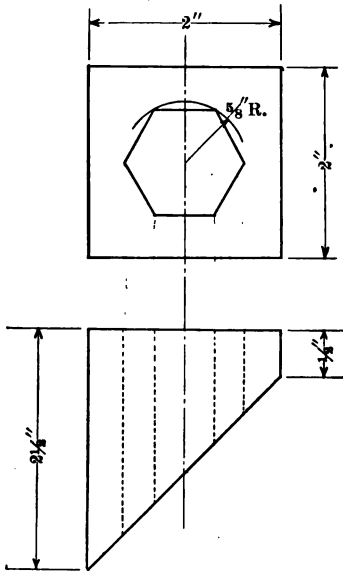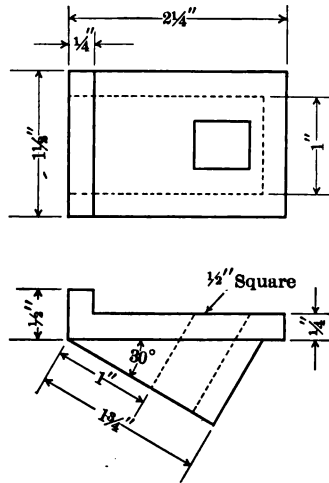
FIG. 16.   FIG. 17.

its edges inclining 30 and 60 degrees to the G. L. Let the edge of the cube nearest V be ½″ behind V.

12. Draw three views of a block 1″ by 2″ by 4″. In the center of one 2″ by 4″ face is a ½″ square hole cut half way through the block, and in the center of each 1″ by 2″ face is a hole ¼″ in diameter drilled 1″ deep.

13. Draw the plan, elevation, and left end view of a bar of iron 4″ long,

2" wide, and ½" thick which has four ¼" holes drilled through it on the center line of the 2" by 4" face. The holes are spaced with their centers ¾" apart. Let the face of the bar be ½" below H and parallel to it, and let the center line of this same face incline 60 degrees to the G. L.

14. Draw three views of a regular truncated hexagonal pyramid. Diameter of large base 3"; diameter of small base 1½"; distance between bases 2". Use first quadrant projection and let the 3" base be in V.
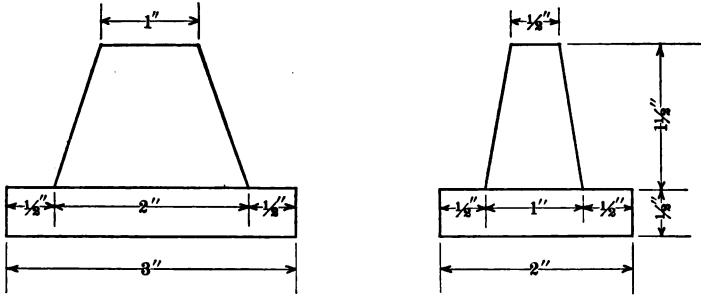
FIG. 18.
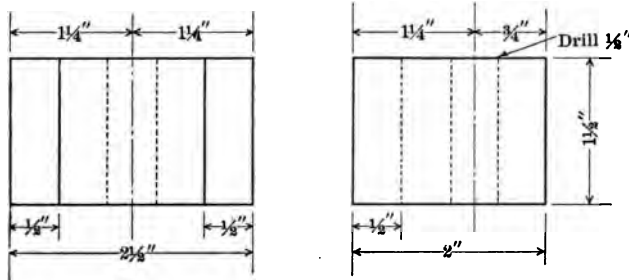
FIG. 19.

15. A hopper has an opening in the floor 4' by 6'. Its 12" square outlet is 5' below the floor with one edge directly under the center of one 6' edge of the floor opening. Draw three views of the hopper.

16. Draw the plan and elevation and find the right end view of the object shown in Fig. 16.

17. Draw the plan and elevation and find the left end view of the object shown in Fig. 17.

18. Draw the two given views and find the plan of the object shown in Fig. 18.

19. Draw the plan of the object whose end view and elevation are shown in Fig. 19.

20. Draw the two given elevations and find the plan of the object shown in Fig. 20.

FIG. 20.

FIG. 21.

Diameter 1"

FIG. 22.

(19)
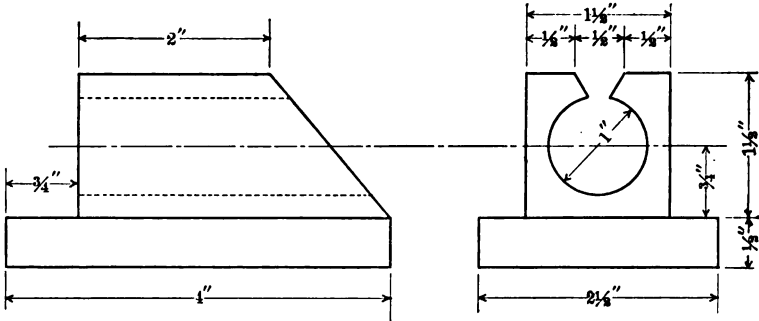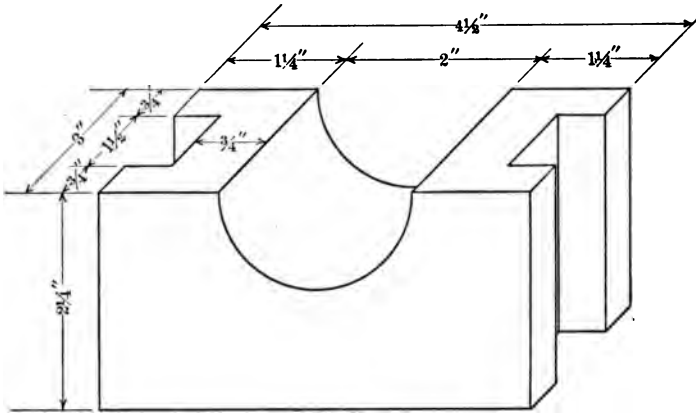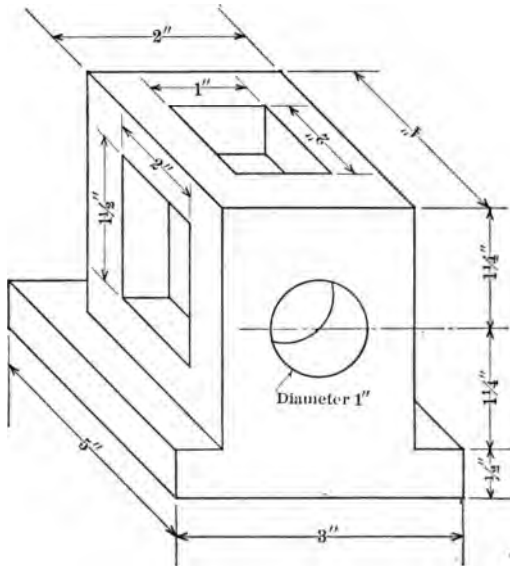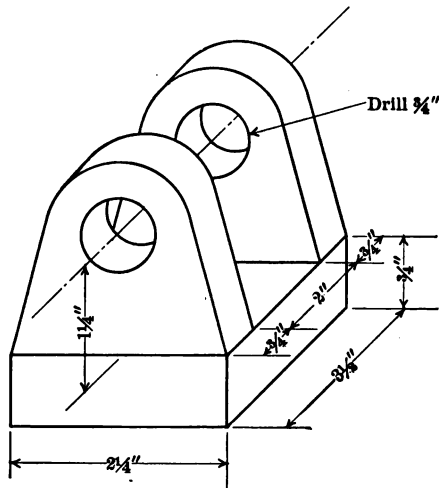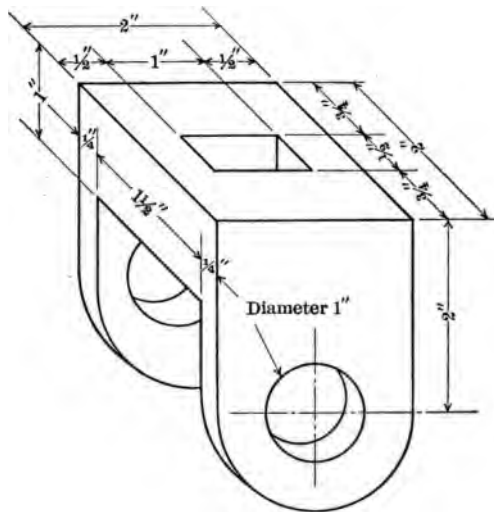
FIG. 23.

FIG. 24.

21. Draw three views of the object shown in Fig. 21.

22. Draw a plan, elevation, and left end view of the object shown in Fig. 22.

23. Draw the plan, elevation, and right end view of the object shown in Fig. 23.

24. Draw a plan, elevation, and both end views of the object shown in Fig. 24.

CHAPTER III

ASSUMPTION OF POINTS AND LINES

15. In order to study the relation of various points and lines which collectively form the representation of an object, and to solve problems concerning these elements of the representation, there must be a method of designating points and lines, and the relation that these points and lines have to the planes of projection must be known.

In drafting a point is almost never designated by a letter, but in descriptive geometry it has been found convenient to indicate points and lines by this means. A point in space is indicated by the capital letter, as A, B, C, etc., and its projections are indicated by the corresponding small letter, as a, b, c, etc.; a, b, c, being used to indicate the plan view of the point and a′, b′, c′, etc., being used to indicate its elevation

FIG. 25.

and a_p, b_p, c_p, etc., being used to indicate the end view. Thus: the plan view of the point A is a, and its elevation is a′, and when the point A is referred to it means the actual point in space. The line MN means a line in space whose plan view is mn, whose elevation is m′n′ and whose end view is $m_p n_p$.
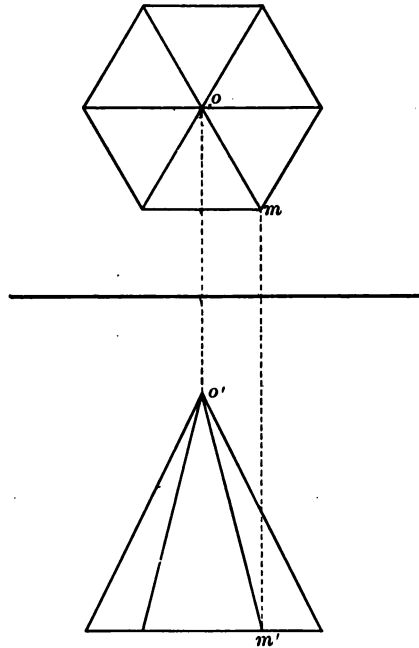
The relation that a point or a line in space has to the planes of projection may be discovered from its projections. In Fig. 25 is shown the plan and elevation of a hexagonal pyramid one of whose edges is OM. Now the plan view of the point O is o and its elevation is o', and an inspection of the figure will show that the distance o is from the G. L. is the distance that O is from the V plane; and the distance o' is from the G. L. is the distance that O is from H. It is also to be seen that since o is above the G. L. the point in space is behind V, and since o' is below the G. L. the point in space is below H. Therefore, from the drawing it may be found that the point O lies in the third quadrant, and its distances from H and V may be measured. In like manner the relation that M bears to H and V may be read from the drawing.

16. To assume a point in space it is necessary only to assume its two projections for, in general, a point in space is completely located with reference to H and V when its plan and elevation are shown. If, however, the problem is to assume a point in a given quadrant at a given distance from H and V, the two views must be located at the proper distances from the G. L. Thus, in Fig. 25, O which is to lie in the third quadrant 1″ from H and 2″ from V will be drawn as follows: Erect the dotted line oo' perpendicular to the G. L.; on this line will lie o and o', the plan and elevation of the required point. Since this required point is the third quadrant it will be below H and behind V; therefore, the plan view o will be above the G. L. a distance equal to the distance the point is behind V, or 2″. Therefore, locate o 2″ above the G. L. on the dotted perpendicular. Since the point is 1″ below H, o' will lie in this same perpendicular 1″ below the G. L.

In like manner a point may be located in any of the quadrants and at any specified distance from H and V.

17. To assume a line in space assume its plan and elevation in any desired position. If two points of the line are given, the plan and elevation of the line may be found by joining the plan views of the two given points, and the elevation by joining the elevations of the two given points. The plans and elevations

of the given points will have to be located as indicated in Article 16.

If the plan and elevation of a line are given and it is desired to assume a point on the line it is necessary only to assume the plan view of the point on the plan view of the line and the elevation of the point on the elevation of the line, and have these two views lie in the same perpendicular to the G. L.

18. To assume two intersecting lines assume two lines which have a common point. Fig. 26 shows AB and BC intersecting at the common point B.

19. There are certain relations which points and lines have to the planes of projection which are apparent from a study of the foregoing articles and these relations may be set forth as, —

Observations.

a. The two views of a point always lie in a perpendicular to the G. L.

b. The distance a point is from H is equal to the

FIG. 26.

distance its elevation is from the G. L.; and the distance a point is from V is equal to the distance its plan is from the G. L.

c. A point which lies in H will coincide with its own plan view and its elevation will be in the G. L. Likewise a point in V will coincide with its own elevation and its plan view will lie in the G. L.

d. A line which lies in H or V will coincide with its view on H or V and will have its other view in the G. L.
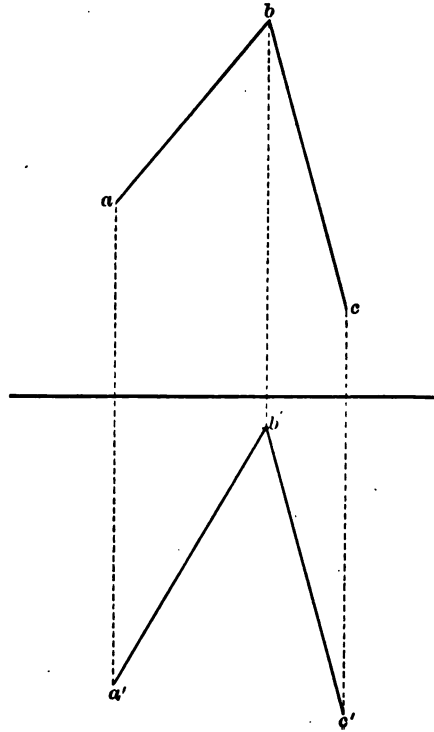
e. When a line is parallel to H or V its view on H or V will be equal in length to the line and its other view will be parallel to the G. L.
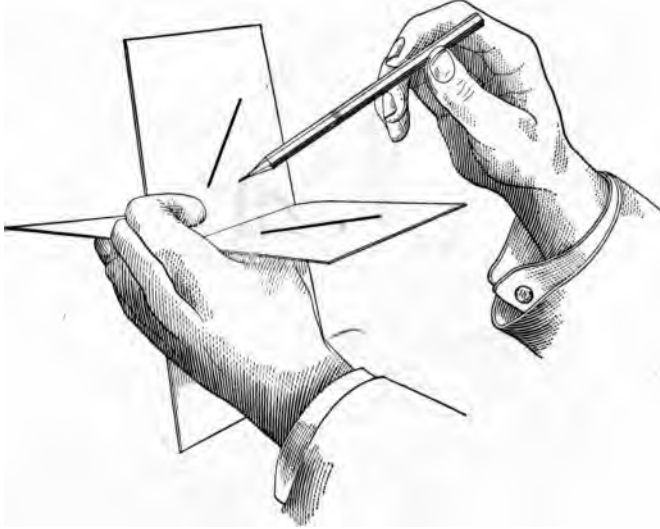
A convenient method of visualizing any given problem in projection. The pencil is held in the position of the given line and its projections on the cardboard planes may be drawn.
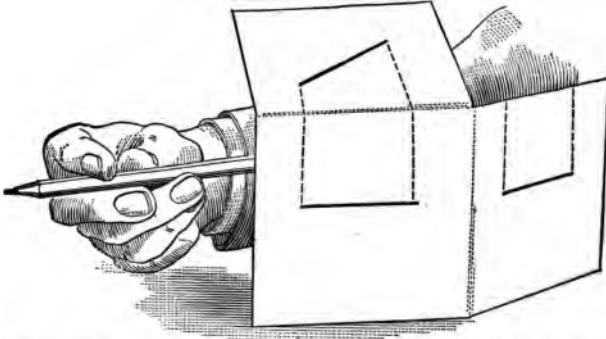
A convenient method of showing the relation of the H, V, and P planes for the third quadrant, and of the projections of any given line on the planes.

f. When a line is perpendicular to H or V its view on H or V will be a point. The other view will be equal in length to the line itself and perpendicular to the G. L.

g. When a line is parallel to the G. L. both views of the line will be parallel to the G. L.

h. The projection of a line on H or V will be equal in length to the line itself when the line is parallel to H or V.

i. The projection of a line on H or V is either equal in length to the line or shorter than the line.

j. A line may lie in one or in two or three quadrants, but never in all four.

k. If a point is on a line the plan view of the point will lie on the plan view of the line, and the elevation of the point will lie on the elevation of the line.

l. When a line is parallel to H and oblique to V the elevation of the line is parallel to the G. L. and the plan view makes an angle with the G. L. equal to the angle the line makes with V.

In like manner when the line is parallel to V and inclined to H its plan is parallel to the G. L. and its elevation makes an angle with the G. L. equal to the angle the line makes with H.

m. When two lines intersect they have a point in common. Therefore, the plan views of these lines will have a point in common, and the elevations will have a point in common.

n. Two lines which are parallel to each other in space will have plan views and elevations which are respectively parallel.

20. These observations should be verified by experiment. A convenient method of doing this which at once serves to verify the experiment and at the same time to assist the experimenter to visualize the problem is shown in the illustrations. Two pieces of cardboard are cut to form the planes of projection and a pencil or a hat pin — which is easier to secure in place — is held in the position the line occupies in space. The projections of this position may be plotted on the cardboard and the planes revolved, thus showing the relation of the plan and elevation of the position to the G. L. These positions should, of course, conform to the positions indicated in the observations.

PROBLEMS IN LOCATION OF POINTS AND LINES

25. Locate a point M 1″ behind V and 2″ below H.

26. Locate a point N ½″ in front of V and 1½″ above H.

27. Locate a point O 1″ behind V and 1½″ above H.

28. Locate a point P 2″ in front of V and 1″ below H.

29. Locate a point M in the third quadrant 2″ from H and 1″ from V.

30. Locate a point N in the first quadrant 1½″ from H and 2″ from V.

31. Locate a point O in the second quadrant 2″ from H and V.

32. Locate a point P in the fourth quadrant 1″ from H and 2″ from V.

33. Assume a line MN in the third quadrant parallel to H and 1″ below H, and making an angle of 30 degrees with V.

34. Assume a line in the first quadrant parallel to V and 2″ from it and inclining to H at 60 degrees.

35. Assume a line in the third quadrant parallel to V and 2″ from V, and making an angle of 30 degrees with H.

36. Draw a line in the first quadrant parallel to the G. L. and 1½″ from H and V.

37. Assume a line with one end in the first and the other end in the third quadrant. Draw an end view of this line.

38. Draw a line parallel to H and making an angle of 30 degrees to V in both first and second quadrants.

39. Assume a point in the third quadrant 2″ from H and 1″ from V. Through this point draw two intersecting lines, one parallel to H and one parallel to V.

40. Assume two intersecting lines in the first quadrant, one parallel to H and one parallel to V and make them 3″ long. Let the point of intersection be 2″ from H and ½″ from V.

41. Assume a line 3″ long in the third quadrant parallel to the G. L. and 1″ from H and V. Through the center of this line draw another line making an angle of 45 degrees with both H and V. Draw an end view of the two lines.

42. One line which inclines 45 degrees to both H and V is intersected by another line which inclines 30 degrees to H and 60 degrees to V. The point of intersection is 2″ from H and V. Draw three views of the two lines in the third quadrant.

43. Draw a line parallel to V 1″ behind it, and making 60 degrees with H. Draw a second line parallel to this line 1½″ from it.

44. A line is 2″ below H and parallel to H. One end is in V and the other end is 3″ behind V, and between these points the line is 4″ long. Through the middle point of this line draw a second line parallel to V.

CHAPTER IV

PLANES

21. A plane is represented in descriptive geometry by its *traces*. These traces are the lines in which the plane intersects H, V, and P, and are designated as the H trace, the V trace, and the P trace.

Since the G. L., which is common to both H and V, can meet the plane in but one point it will be obvious that the H trace and the V trace must meet each other at this point. In designating planes on a drawing this point is marked S, T or R, etc., while the H trace is marked with the corresponding small letter s, t, or r, etc.; and the V trace s', t', or r', etc.; and the P trace s_p, t_p, or r_p, etc.

In drafting, planes are usually so arranged as a part of the object being drawn that they are represented by limiting lines. For example in a cube the faces are shown in plan and elevation as squares and the traces of the planes of the faces are not needed. But while the planes of any face of an object are usually not needed to make the complete representation, yet they are useful in solving drawing-board problems which arise in connection with the drawing of the objects. They should, therefore, be studied with the idea in mind that they are a means to an end rather than an end in themselves. For this reason it will be convenient to represent traces with a dashed line arranged as shown in the figures following.

22. Planes in descriptive geometry are considered to be indefinite in extent. For this reason, then, it should be clear that any plane extends through all four quadrants even though its traces show it as being in only one. It is customary to show the traces of a plane only in that quadrant where the problem lies, yet if it were necessary these traces could be extended.

In Fig. 27 is shown a plane in the first quadrant; in Fig. 28 a plane in the second quadrant; in Fig. 29 a plane in the third quadrant; and in Fig. 30 a plane in the fourth quadrant.   Fig.
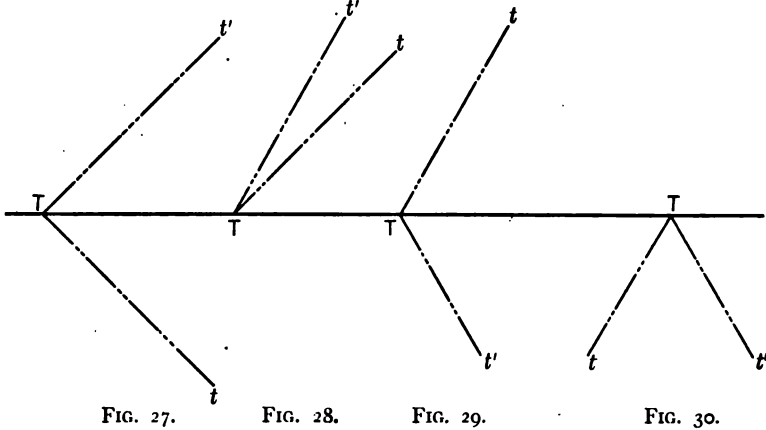
FIG. 27.   FIG. 28.   FIG. 29.   FIG. 30.
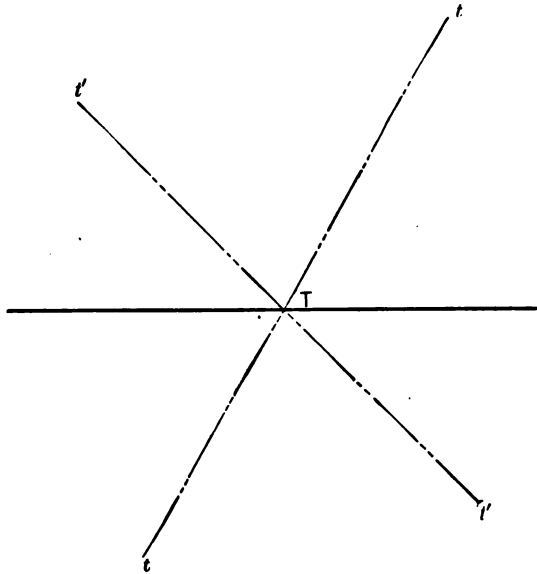
FIG. 31.

31 shows a plane indefinite in extent and therefore in all four quadrants: it is obvious from this figure that any pair of the given H or V traces will locate this same plane.

23. Observations.

a. The traces of a plane which is parallel to the ·G. L. will themselves be parallel to the G. L. Fig. 32 shows such a plane.

b. If two planes are parallel their respective traces will be parallel.

c. If a plane is perpendicular to H, its V trace will be perpendicular to the G. L. Likewise, when it is perpendicular to V its H trace will be perpendicular to the G. L.

S ————··—————··—————··——————··———*s′*

S ————··—————··————··——·——··————*s*
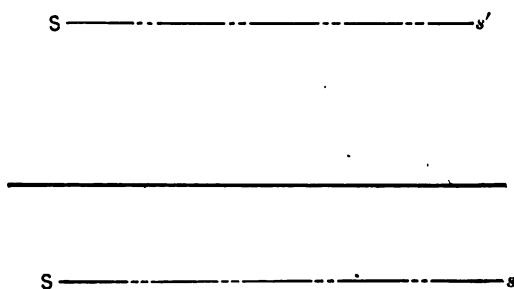
FIG. 32.

d. If a plane is perpendicular to H it will also be perpendicular to any other plane whose H trace is perpendicular to its H trace. Likewise a plane perpendicular to V will be perpendicular to any other plane whose V trace is perpendicular to its V trace.

e. If the plan and elevation of a line are perpendicular respectively to the H and V traces of a plane the line will be perpendicular to the plane.

f. If a line is perpendicular to a plane, the plan view of the line will be perpendicular to the H trace of the plane and the elevation of the line will be perpendicular to the V trace of the plane.

g. A plane is determined by any two intersecting lines, any three points, any two parallel lines, or a point and line. The plane is, of course, in each case represented by its traces.

h. The plan view of the V trace of any plane is in the G. L.; and the elevation of the H trace of any plane is also in the G. L.

i. If a line lies in a plane and is parallel to the H trace its plan view will be parallel to that trace and its elevation will be parallel

to the G. L.   Likewise, when the line is parallel to the V trace
its elevation will be parallel to that trace and its plan will be
parallel to the G. L.

These observations are collected here for convenience and
reference.   No attempt has been made to prove the truth of the
assertions set forth because they are more or less axiomatic, and
both the proof and truth of these facts will be obvious as the
study of the subject
progresses, and may be
proved by experiment
as in Article 19.
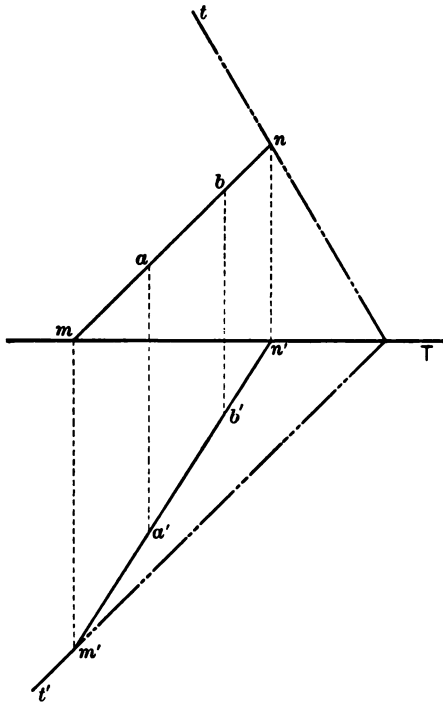
**24. To assume a
plane,** assume any two
traces which are parallel
to the G. L., or which
intersect the G. L. at a
common point.

**25. To assume a line
in a plane,** assume either
view of the line, as the
plan view ab in Fig. 33.
The plan view when ex-
tended cuts the G. L. at
m and cuts the H trace
of the plane at n.   Since
n is on the H trace its
elevation will be on the
G. L. at n'; and since m
is on the G. L., m' —

FIG. 33.

since the line AB is to be in the plane T — must lie on the V
trace.   m'n', therefore, is the elevation and mn the plan of MN
lying in the plane T.   ab and a'b' are, of course, on mn and
m'n' and are the plan and elevation of a line AB lying in the
plane T.

26. To assume a point in a plane. —One method is to assume
a line in the plane — as in the foregoing article — and on this
line assume the point.   Another method more convenient to use

is to assume one view of the point — as the plan view a in Fig.
34 — and through this assumed plan view of the point draw the
plan mn of a line MN parallel to the H trace.   Since this line
MN is parallel to the H trace its elevation will be parallel to the
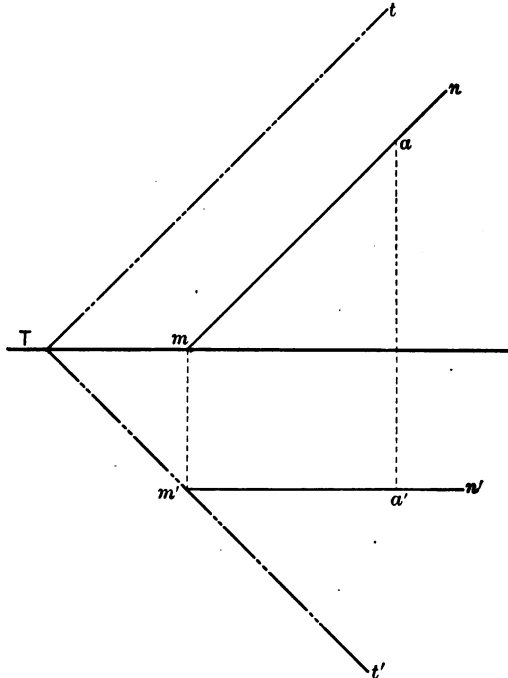
FIG. 34.

G. L.; mn and m'n', then, are the plan and elevation of a line
MN lying in the plane T.   Since a lies on mn, a' will lie on
m'n', and the point A then, as represented by the plan a and
elevation a', will be a point in the plane T.

PROBLEMS ON PLANES

45. Assume a plane parallel to the G. L. so that its H trace is 2″ back
of V and its V trace 3″ above H.   Show an end view of this plane.

46. A plane is perpendicular to H and inclined 30 degrees to V.   Show
its three traces.

47. Draw a plane parallel to the plane in Problem 46.   Draw the profile
traces 2″ apart.

48. Draw the H, V, and P traces of a plane in the third quadrant. Let the H trace incline 60 degrees to the G. L. and the V trace 45 degrees to the G. L.

49. Assume any line in the first quadrant oblique to H and V and show. the traces of a plane perpendicular to this line.

50. Assume any oblique plane in the third quadrant and show the traces of a plane which will be perpendicular to this plane and to V.

51. Assume a plane whose H and V traces incline 60 degrees to the G. L. In this plane is cut a hole whose plan view is a 1″ square. Assume the location of the plan view of the hole and show its plan, elevation, and end view.

52. Assume a plane whose H trace makes 45 degrees with the G. L. and whose V trace makes 60 degrees with the G. L. In this plane is cut a hole whose elevation is a circle 1″ in diameter. Find also the plan and end view of this hole. The location of the elevation may be assumed.

53. A plane inclines 60 degrees to V and 30 degrees to H and is parallel to the G. L. 3″ from it. In this plane is cut a hole 1″ square; show its plan, elevation, and end view.

54. A plane inclines 60 degrees to V and is perpendicular to H. In this plane is cut a 1″ circular hole with its center 1½″ below H. Show three views of the plane with the hole cut in it.

CHAPTER V

LOCATION OF POINTS, LINES, AND PLANES

27. Character of Lines. In order to facilitate the reading of the drawings in the text all given and required lines will be drawn heavy and all auxiliary and constructive lines will be drawn light. When necessary the two views of a point will be connected by a light dotted line perpendicular to the ground line, but wherever possible the line is omitted to avoid the confusion of unnecessary lines. The following chart shows the character and purpose of

———————————————————— Visible outline

-- Invisible outline

—— ·—— ··—— ··—— ··—— ··——— Trace of plane

— — — — — — ·—— — — — — — — — Path of moving point

--- Joining projections of point

—— ·—— ·—— ·——— ·—— ·—— ·—— Center line

CHART OF LINES

each line used in the drawings and will be found convenient for reference.

28. Method of Locating Points and Planes. — A point will be located by giving its distances, or coördinates, in inches always unless otherwise marked, from P, V, and H and *in that order.* Thus: A is o; 2; 2 means that A in space is in the profile plane, two inches from V, and two inches from H; or B is 3; 4; 1 means that B is three inches from the profile plane, four inches from V, and one inch from H.

In order to indicate on which side of the planes of projection a point lies, — or, in other words, in which quadrant it is, — plus and minus signs will be used. Plus always means to the right of P, in front of V, and above H, and unless otherwise marked points will be understood to be plus. Thus A is 4; 3; 2 means that A is four inches to the right of the profile plane, three inches in front of V, and two inches above H. In other words, it is in the first quadrant.

Minus always means to the left of P, behind V and below H. Thus: B is —2; —3; —5 means that B is two inches to the left of P, three inches behind V, and five inches below H. B, in other words, is in the third quadrant. It will be convenient to remember that a plus sign preceding the last two figures means that the point is in the first quadrant. Thus M is —4; +2; +3 means M is in the first quadrant four inches to the left of P, two inches in front of V, and three inches above H. N is 2; —1; —2 means that N is in the third quadrant two inches to the right of P, one inch behind V, and two inches below H.

A line will be located by giving reference points for any two of its points but it is to be understood that these two points do not necessarily limit the extent of the line. Thus M is —2; —3; —4; N is 3; —4; —2 means that the line MN is in the location indicated by the location of M and N, but it is not necessarily limited in length by M and N.

A plane will be located by giving first, the distance its point on the ground line is from P; second, the angle its V trace makes with the ground line; and third, the angle its H trace makes with the ground line. The intersection of the traces on the ground line is the vertex of these angles. To indicate the direction of the traces plus and minus signs will be used as follows: a plus sign preceding the angle given for the V trace means that the V trace is *below* the ground line, and a minus sign means that the V trace is *above* the ground line; a plus sign preceding the angle given for the H trace means that the H trace is *below* the ground line, and a minus sign means that the H trace is *above* the ground line. Measurement of these angles begins always on the ground line at the right of the

vertex and *plus* angles for both traces are measured *clockwise* while *minus* angles for both traces are measured *contra-clockwise*. The examples given below illustrate this method of locating planes.

S — 0″ — 60°; + 45°   S — 0″; + 45°; –120°   S — 0″ — 60°; + 90°   S — 0″; –150°; + 120°
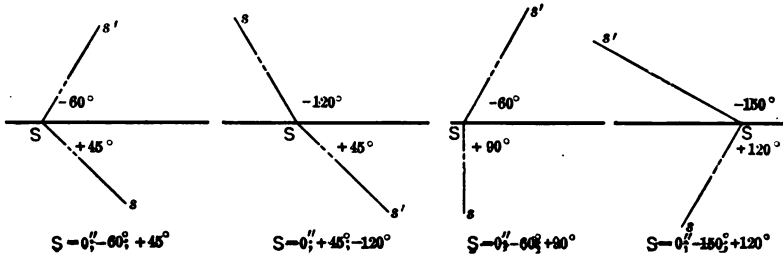
Since planes are indefinite in extent the values of the angles given in the problems often exceed 180 degrees, as $T = +2''$, $-300°$, $+300°$. In such cases 180° may be subtracted from the given values and the method given above will apply. $T = +2''$, $-300°$, $+300°$ would then become $T = +2''$, $-120°$, $+120°$.

When a plane is parallel to the ground line the traces make no angle with the ground line. Such planes are located by giving first, the symbol ∞ (which means that the plane is parallel to the ground line and its traces meet at infinity); second, the distance the V trace is from the ground line; and third, the distance the H trace is from the ground line. A plus

S—∞; +1″; +¾″   S—∞; –1″; –1¼″   S—∞; +1″ –½″
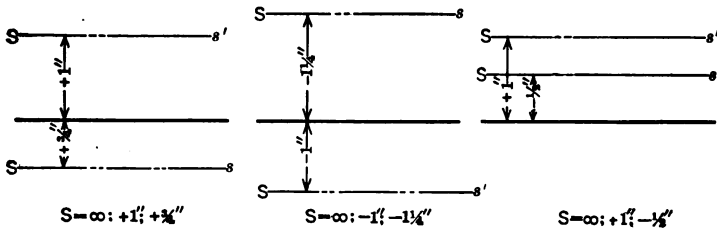
sign preceding the distance given for the V trace means that this trace is *above H*, and a minus sign means that this trace is *below H;* a plus sign preceding the distance given for the H trace means that this trace is *in front of V*, and a minus sign means that this trace is *behind V*. The examples given above

illustrate this method of locating planes parallel to the **ground line.**

29. Accuracy and Checking. — In working the problems on the drawing board it is reasonable to expect results within two hundredths of an inch. That is, if a line is actually two inches long its length should be found within the limits one and ninety-eight hundredths and two and two hundredths inches. Such a degree of accuracy means, of course, great care in drawing and plotting. An error made at the beginning of a problem is apt to accumulate as the problem progresses, and the only way to get close results is to check the problem step by step.

Checking means verifying a result by another method, and it is important in drafting operations that this means of verifying work be resorted to often.

PROBLEMS ON THE LOCATION OF POINTS, LINES, AND PLANES

55. A point is in the third quadrant $3''$ to the left of the profile plane, $2'$ from H and $4''$ from V. Write these distances in the proper order, and with the correct signs. Show also three views of the point.

56. The point M is $3''$ above H, $2''$ in front of V, and in the profile plane. Show three views and give the coördinates of the point.

57. Locate the following points and indicate in what quadrant each is:

$$M = 0; \ 2''; \ 1''$$
$$N = -1''; \ -1''; \ 2''$$
$$O = -2''; \ -1\tfrac{1}{2}''; \ -2\tfrac{1}{2}''$$
$$P = -3''; \ 2''; \ -1''.$$

58. M $= 0; \ -2''; \ -1''$. N $= +2; \ -1''; \ -\tfrac{1}{2}''$. O $= 1''; \ -2''; \ -2''$. From O draw a line OP to the middle point of the line MN and give its coördinates.

59. A $= -3''; \ +2''; \ +1''$. B $= -1''; \ +4''; \ +2''$. C $= -\tfrac{1}{2}''; \ +1''; +1''$. Draw the plan, elevation, and end view of the triangle ABC.

60. M $= +2; \ -1; \ -1$. N $= -2; \ +2; \ +2$. Draw a plan, elevation, and end view of this line. Use a profile plane at -3 for the end view.

61. A $= 0; \ -2; \ +1$. B $= 0; \ +2; \ -1$. Show a plan, elevation, and end view of this line.

62. M $= 0; \ -3; \ 0$. N $= 0; \ 0; \ -3$. Show a plan, elevation, and end view of this line and from the G. L. erect a perpendicular to it. Give the coördinates of the point of intersection.

63. A $= -3; \ -2; \ -1$; B $= -1; \ -2; \ -4$. With AB as one side construct a square parallel to V and show three views of it.

64. M = o; −3; −2; N = +3; o; +2; O = +1; −1; o.   Draw three views of the triangle MNO.

65. Show three views of the solid — one of whose faces is ABCD — whose corners are given below:

A = o; −2; −2; B = +2; −2; −2; C = +2; −2; −4; D = o; −2; 4;
E = o; −4; −4; F = o; −4; −2; G = +2; −4; −2; H = +2; −4; −4.

66. A = +2; −3½; −2; B = +2; −1½; −2; C = +2; −2¼; −3¼; O = +6; −2¼; −2¾.   Draw three views of this pyramid considering ABC as the base and O the apex.

67. A cube 2″ on each edge has one corner at +2″; −3″; o.   The edge, of which this point is one end, slopes away from H at an angle of 30 degrees to the left and is parallel to V.   Draw three views of the cube in the third quadrant.

68. Locate the three traces of the plane o; −30; +60.

69. Locate the three traces of the plane −2; +60; −45.

70. Locate the three traces of the plane +1; +60; +30.

71. Locate the three traces of the plane o; −45; −75.

72. Locate the three traces of the plane o; −135; +45.

73. Locate the three traces of the plane o; −150; +105.

74. Locate the three traces of the plane −1; −165; +150.

75. Locate the three traces of the plane +2; −90; +90.

76. Locate the three traces of the plane −2; +90; −75.

77. Locate the three traces of the plane −3; +135; −135.

78. The plane S = o; −45; +60.   Assume a line in this plane 2″ above and parallel to H.

79. The plane T = −2; −135; +120.   Assume a line in this plane 1″ behind and parallel to V.

80. The plane S = o; +60; −60.   Assume a line in the plane 2″ below H and parallel to H.

81. The plane S = −2″; +120; −135.   Assume a line in this plane parallel to V and 1″ behind V.

82. Locate the planes

$$\infty; \ -3; \ -4$$
$$\infty; \ -3; \ \infty$$
$$\infty; \ \ \infty; \ +3$$
$$\infty; \ \ \infty; \ -2$$
$$\infty; \ +2; \ -1.$$

83. The plane S = ∞; +3″; −4″.   Find the coördinates of a line 3″ long in this plane 2″ from H.

84. The plane T = ∞; +2″; −3″.   Draw a hole 1″ square in this plane, and show three views of it.

CHAPTER VI

REVOLUTION OF POINTS

30. One of the most important operations in descriptive geometry and the means by which a great many problems are solved is the revolution of a point, or a line, from one position to another. The actual laws governing such a movement are well known, but to represent the change of position correctly, and to visualize, or to get a mental picture of what is taking place during the change of position, requires careful study.
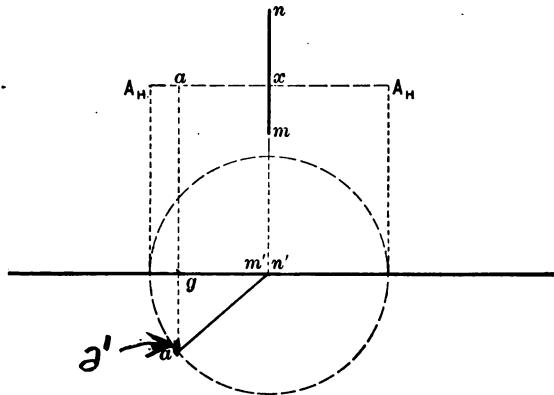
FIG. 35.

31. When a point revolves about a line two things are apparent at once:

1. That the point describes a circle in space.

2. That the plane of this circle is perpendicular to the line.

Put in other words, the revolution of any point about an axis takes place in a plane perpendicular to the axis.

32. Fig. 35 shows the plan and elevation of a circle which has been made by revolving the point A about the line MN which lies in H. From this figure it will be seen that when A lies in H,

or the plane of the axis, it will be at A_H and that the distance from A_H to the axis is equal to the radius of the circle.

In terms of the distances which are shown by the plan and elevation of point A, this radius is equal to the hypotenuse of a right triangle whose base is equal to the distance a is from the plan of the axis, or ax, and whose altitude is equal to the distance a' is below the G. L., or the distance A is below the plane of the axis, or a'g. Since the plane of revolution is perpendicular to the axis, A will fall on a perpendicular through a to mn,

FIG. 36.

either on the right or the left of MN, a distance from MN equal to a'n', or the hypotenuse of a triangle whose base is ax and whose altitude is a'g.
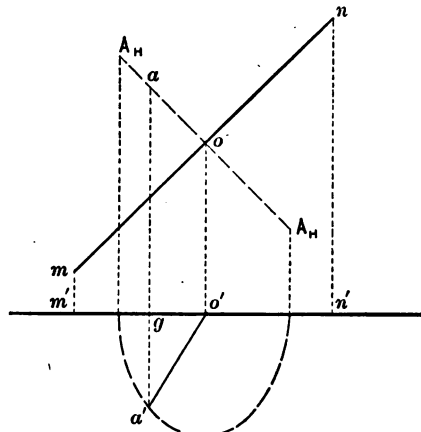
33. Fig. 36 shows this same problem worked out when the axis MN lies in H but oblique to V. In this case A_H will be found on a perpendicular from a to mn whose length is equal to the hypotenuse of a triangle whose base is ao, and whose altitude is the distance from a' to the G. L., or a'g.
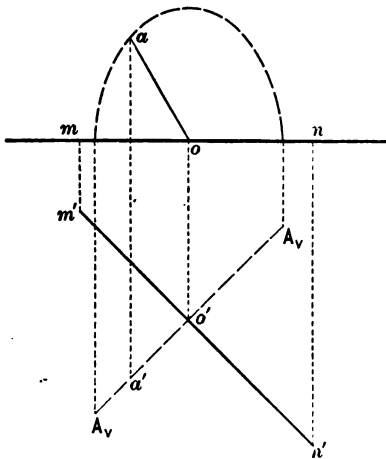
Fig. 37 shows a similar problem worked out with the point revolved into V about an axis which lies in V. In this case the point A falls on a perpendicular to m'n' whose length is equal to the hypotenuse of a triangle whose altitude is equal to

FIG. 37.

the distance a is from the G. L., and whose base is the distance a'o'.
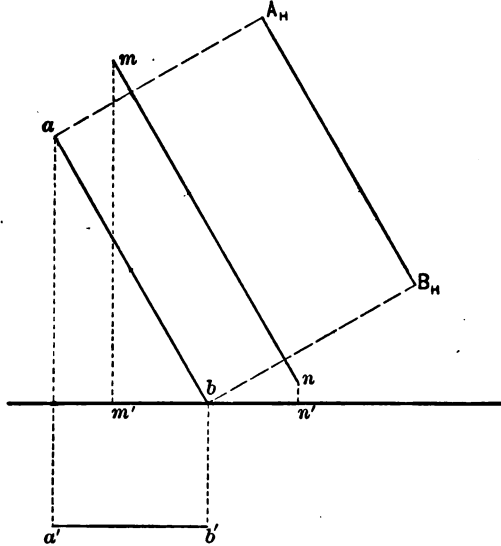
FIG. 38.

34. A line may be revolved into H or V about an axis which lies in H or V by revolving any two of its points. The line and

FIG. 39.

axis must, however, lie in the same plane else one point of the revolving line would touch H or V before the other. Fig. 38 shows a line AB revolved into H at $A_H B_H$ about MN which lies in H and is parallel to AB. The location of the points A_H and B_H may be found as indicated in Article 32.
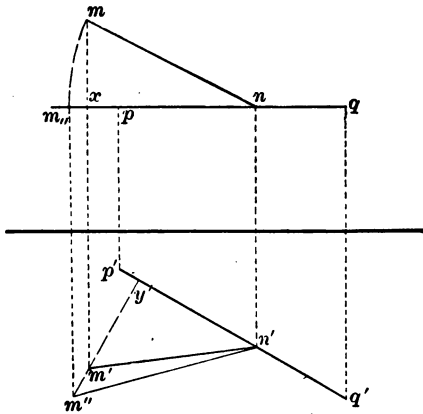
35. Either a point or a line may be revolved about an axis into the plane of the axis when the plane does not coincide with the H or V but is parallel to H or V. The method by

which this may be done may be readily understood by considering that a secondary H or V plane passes through the axis. The revolution, then, may be referred to this new H or V plane and may be accomplished as indicated in Article 32.

Fig. 39 shows the plan and elevation of a line MN which is to be revolved into the plane of the axis PQ. The revolution may be accomplished as indicated above or by working the
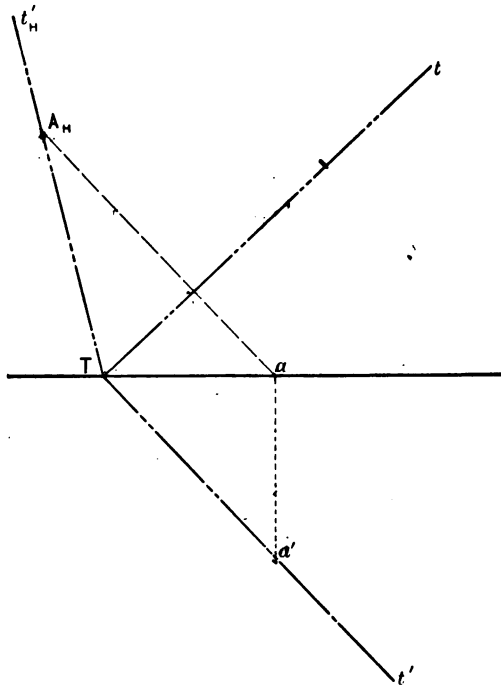
FIG. 40.

problem without the use of the secondary H or V plane. In the figure the line MN has been revolved into the plane PQ directly, M falls at a point whose elevation is m'', m'' being a distance from the axis equal to the hypotenuse of a triangle whose base is the distance of m' to the axis, or $m'y'$, and whose altitude is the distance M is from the plane of the axis, or mx.

36. To revolve a plane into H or V about the H or V trace. Assume a point on the trace, as A on the trace $t'T$ in Fig. 40.

Revolve this point into H about tT as an axis as in Article 32. Through this revolved position, A_H, and the point T on the G. L. draw the line $t'_H T$. This will be the revolved position of the V trace as it lies on H and $t'_H Tt$ is the revolved position of the plane T.

To revolve the plane into V a point on the H trace should be taken and the V trace used as an axis.
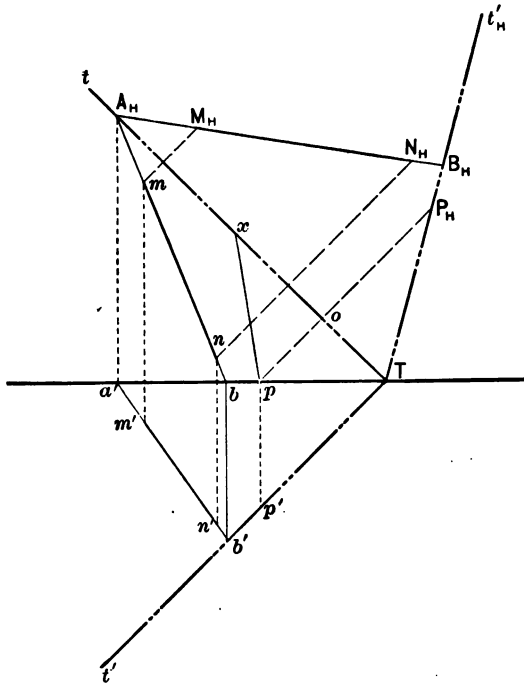
FIG. 41.

37. To find the plan and elevation of a line in a plane when the revolved position of the line and plane are given.

In Fig. 41 is shown a plane T and a line MN in the plane T as they lie revolved into the H plane at tTt'_H and $M_H N_H$. To find the actual position of the traces and the plan and elevation of the line assume P_H, the revolved position of a point on the revolved V trace. P, since it is on the V trace, will lie in V and its plan view therefore lies on the G. L. Through P_H draw a per-

pendicular to Tt, since Tt must be the axis of revolution, and extend it to the G. L. at p. p, then, is the plan of a point P on the V trace, and the elevation will be at p′ which is below the G. L. a distance equal to ox. To find ox lay off px equal to op_H. ox then is the base of the triangle whose hypotenuse is px, or oP_H. Draw t′T through T and p′ and it will be the true position of the V trace and tTt′ will represent the true position of the plane.

To find the plan and elevation of any line MN in this plane whose revolved position is $M_H N_H$ extend $M_H N_H$ to A_H on the H trace and B_H on the revolved V trace. When the plane is revolved back to its true position tTt′, B_H will fall at b′ so that Tb′ is equal to $B_H T$ and A_H falls at A_H. A is shown in elevation at a′ on the G. L. and B is shown in plan at b, also on the G. L. A_Hb is the plan and a′b′ is the elevation of the required line extended, and M and N may be located on this line.

PROBLEMS ON REVOLUTION

85. M = o; +1; o.　N = +3; +4; o.　O = +1; +2; +2. Revolve O into H using MN as an axis. Find the length of the radius of revolution.

86. A = o; o; +1. B = +3; o; +4. O = +2; −1; −1. Using AB as an axis revolve O into V. Give the length of the radius of revolution.

87. M = o; −2; −2. N = +3; −1; −1. Revolve MN into H about its own plan view.

88. A = −3; o; o. B = +1; −2; −2. Revolve AB into V about a′b′ as an axis.

89. A = +1; −2; −o. B = −2; −1; −3. Revolve AB into H about ab.

90. The plane S = −2; +30; −45. Revolve plane S into H about sS.

91. The plane T = +1; +120; −135. Revolve plane T into V about its V trace.

92. The plane S = o; +45; −60. Revolve this plane into H and in this position locate a 1″ square hole in this plane. Show the plan, elevation, and end view of the hole after the plane is returned to its original position.

93. The point T = o; o; o. The point t′ = +3; o; −2. The point tv = o; o; −3. t′Ttv is the revolved position of a plane T as it lies in V. Show its true position.

94. The point S = o; o; o. The point s = +2; −3; o. The point s′H = −2; −3; o. sSs′H is the position of a plane S as it lies revolved into H. Show it in its true position.

95. The plane $T = \infty$; -2; -3. The point O which lies in T lies at the point o; x, -1. Find the plan and elevation of the point and revolve into V about $t'T$.

96. Plane $S = o$; -60; $+30$. The point O lies in this plane at -2; -1; x. Show a plan and elevation of this point and revolve it into H about sS.

CHAPTER VII

PROBLEMS ON THE LINE

38. Proposition 1. Given two views of a line to find its length.

Discussion. Revolve the line parallel to any of the planes of projection. In this position the line will be projected on the plane to which it is parallel in its true length. Therefore, the length of the projection is the length of the line itself.

Construction. First Method. Let it be required to find the over-all length of the brace AB shown in the drawing of the bracket in Fig. 42. Using the center line of the vertical post as an axis turn the brace until it becomes parallel to the V plane and takes the position shown by the light lines. Both A and B will describe arcs of circles during the revolution, and since the planes of these arcs are perpendicular to the axis of the revolution (Art. 32), the points will remain the same distance below H. In the new position shown by a'' and b'' the line is parallel to V and the distance $a''b''$ is the true length of the brace.

Construction. Second Method. In Fig. 43 the same problem is solved by revolving AB into H by using ab as an axis of revolution. The revolved position of A is found at A_H by Article 32. A_Ha is equal to the distance from a' to the G. L. and is drawn perpendicular to ab. In like manner B is found at B_H. The line A_HB_H being parallel to H is, of course, equal in length to AB.

Corollary. Given the length of a line and one view of it to construct the other view, when the position of some point on the line is known.

Construction. Let the problem be to make a drawing of the hopper (Fig. 44) whose plan is abcd — mnop. The length of the edge BN is $12''$, and the plane of the opening ABCD lies $4''$ below H.

First draw the line a′b′c′ 4″ below the G. L.; this is the eleva-
tion of the opening ABCD. Now with ·b′ as a center strike an
arc whose radius is 12″; somewhere in this arc will lie n″, the
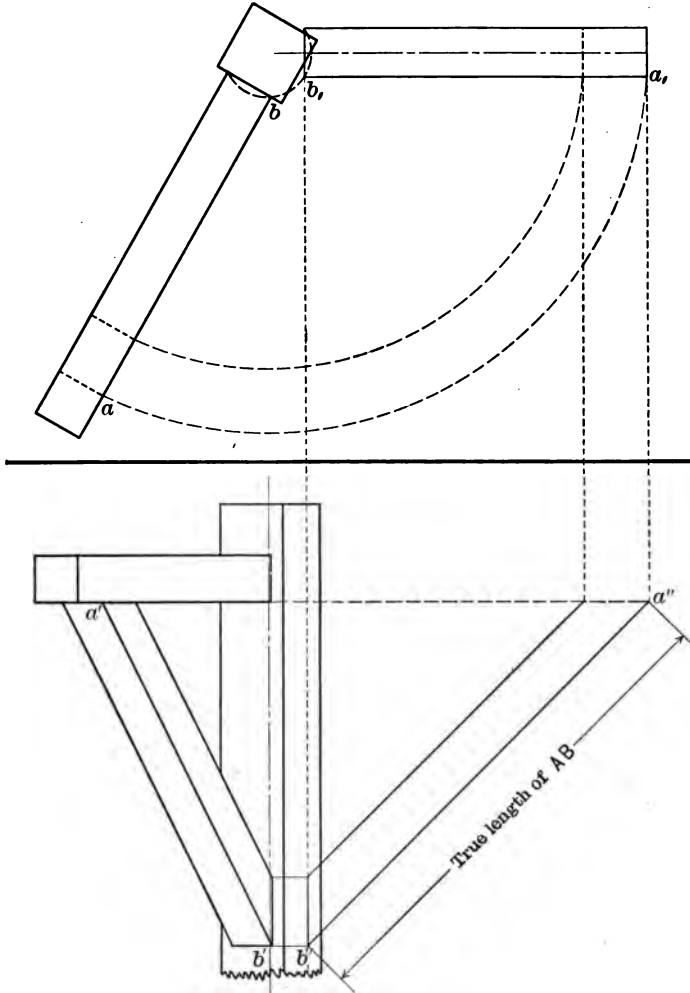
FIG. 42.

elevation of the revolved position of N. To find this point re-
volve BN parallel to G. L. at bn,. From n, drop a perpendicu-
lar to intersect the arc; the point thus found is n″, the elevation
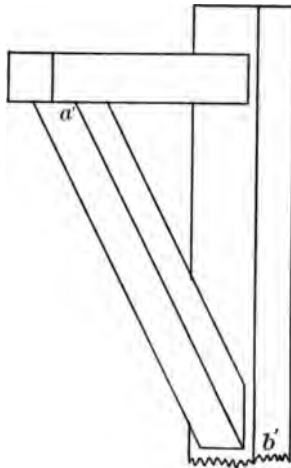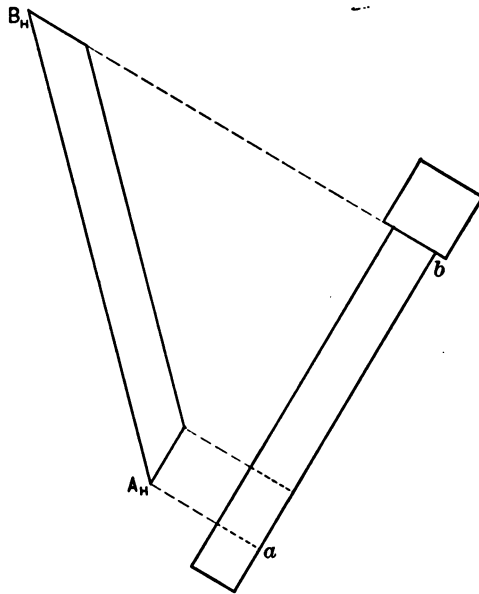
FIG. 43.

of the revolved position of N. BN may now be revolved to the first position bn, and n″ will be shown at n′. The elevation of the hopper may now be drawn as shown by the figure.

39. Special Cases. 1. To find the length of a line which lies in the first and third quadrants.
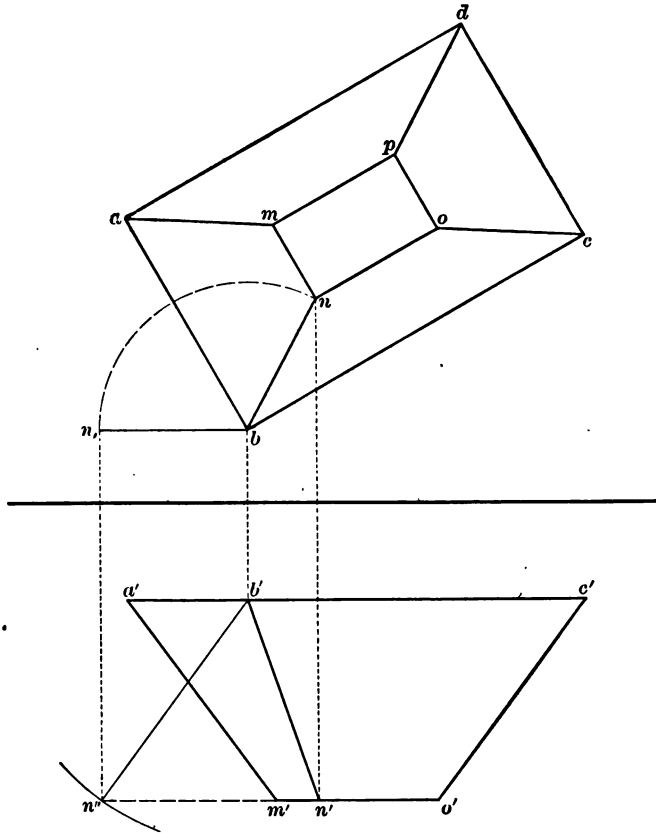
FIG. 44.

2. To find the length of a line which lies in the profile plane.

40. Proposition 2. Given two views of a line to find where it pierces the planes of projection.

Discussion. The point where the line pierces H must lie on the line itself and in the H plane. Therefore, the projections of the required piercing point will lie on the projections of the given

line, and the elevation will lie on the G. L.   If the elevation of
the line be extended to meet the G. L. this point will be the ele-
vation of the required piercing point; its plan view will lie on the
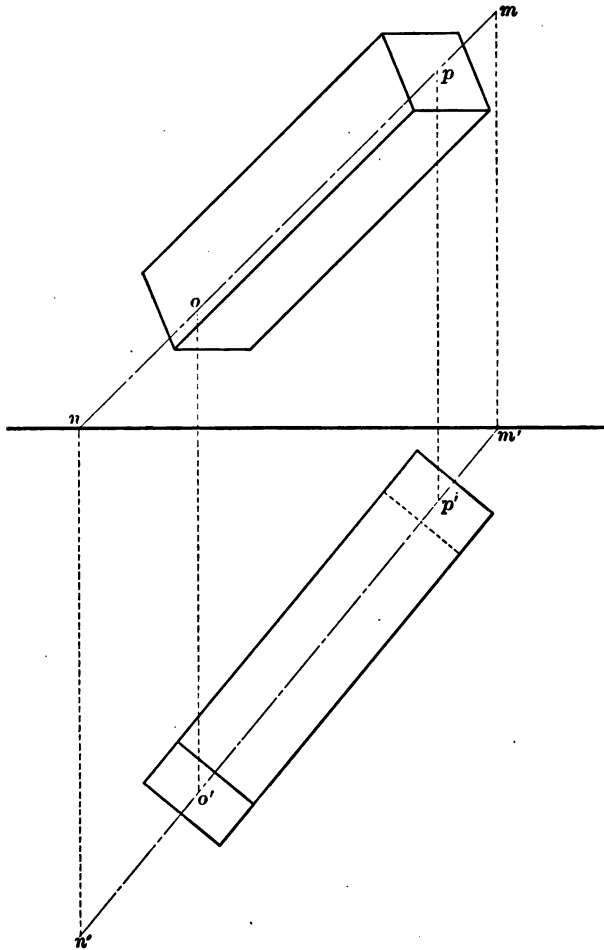
FIG. 45.

plan view of the line and in a perpendicular from the elevation.
In a similar manner the V piercing point may be found.

   Construction.   In Fig. 45 let the problem be to find the plan
and elevation of the point where the center line OP of the timber
pierces H and V.   By extending o'p' to the G. L. the point m'

is found; m′ is the elevation of the point where the line OP pierces the H plane.   The plan view of this H piercing point will be on op where the perpendicular from m′ intersects op at m; since M is in H (as shown by the elevation m′) and on OP (as m and m′ show) it will be the point where OP pierces H.

To find where OP pierces V, op is extended to the G. L. at n, and n′ is located on o′p′ as shown.   The point N, being in V and on the line OP, will be the point where OP pierces V and n is the plan and n′ the elevation of this point.

In a similar manner the P piercing point, if it be required, may be found.

41. Special Cases. 1. Given the location of the H and V piercing points of a line and the length of the line to draw its plan and elevation.

2. Find the piercing points of a line which lies in a profile plane.

PROBLEMS ON THE LINE

97. Draw the plan and elevation of a triangular pyramid whose base is A = 0; −1; −1;   B = +1; −3; −1;   C = +1½; −2; −1,   and whose apex is O = +1; −2; −4.   Find the true length of OA.

98. The center line of a pipe intersects the floor of a room at the point A = −36″; +48″; +0″, and intersects the front wall at B = −36″; 0″; +36″.   Find how long the pipe is between these points.

99. The center line of a timber is so located that one end is at the point M = 0; +36″; +6″, and the other end is at the point N = −48″; −36″; −36″.   Find how long the timber is.

100. A brace rests against a floor at 0; −24″; 0, and against the wall at −36″; 0; −48″.   Find the true length of the brace (center line only considered) between these points.

101. The center line of a tunnel is 500′ long.   One portal lies at the point 0; −0′; −40′, and from this point the tunnel runs north 60 degrees east on a rising 6 per cent grade.   Show its plan and elevation and find the actual distance between portals.

NOTE.   In all land measurement distances are measured on the horizontal.   Thus, the length of the tunnel given as 500′ is really the length of its H projection or plan view, and the distance asked for is the actual linear distance from one portal to the other.

102. A drain starts at a point −48″; −36″; 0, and runs north 45 degrees east for 96″ on a falling 10 per cent grade.   How many inches of drain pipe will be required?

103. A prop 10' long rests on a floor 6' in front of wall. Find where it will rest against the wall when its plan view inclines 60 degrees to the wall.

104. A pipe lies in a side wall so that it slopes 30 degrees to the horizontal and goes through the ceiling 6' back of the front wall. Find where it pierces the front wall.

105. A tunnel starts from the bottom of a shaft at 0; − 100'; −65', and runs south 60 degrees east on a rising 15 per cent grade. Find the point where the tunnel will reach the surface and how long it will actually be.

106. A brace rests against a wall 60" above the ground, and rests against the ground 60" in front of the wall. The brace is 108" long. Show three views of it.

107. How many feet of pipe will be required to join an opening in a floor 18" back of the front wall of a building with an outlet in the left side wall 60" below the floor and 36" back of the front wall? The opening in the floor is 48" from the side wall.

CHAPTER VIII

PROBLEMS ON THE PLANE

42. Proposition 3. To find the traces of a plane when two views of any of its lines are given.

Discussion. Since the trace of a plane is the line in which the plane cuts H or V it must contain the points in which all lines contained in the plane cut H or V. Therefore, if the H and V piercing points of the given lines be found these will determine the traces of the plane of the lines.

Construction. Let it be required in Fig. 46 to find the traces of the plane of that side of the hood given by the two lines AB and BC. By Proposition 2 find where AB and BC pierce H and V; AB piercing H at M and V at N; BC piercing H at O and V at P. Join M and O; this will give the H trace tT. Join N and P; this gives the V trace t'T. The plane of the two lines then is tTt'. The traces must of course meet on the G. L.

43. Special Cases. Find the traces of the plane of two lines:

1. When one line is parallel to H.
2. When one line is parallel to H and the other parallel to V.
3. When one line is parallel to the G. L.
4. When one line is parallel to the G. L. and the other is parallel to H or V.
5. When the two lines are given by their plans and end views.

NOTE. In working the above special cases it should be remembered that when a line lies in a plane and is parallel to H or V, it will be parallel to the H or V trace of the plane. See observations on planes, Article 23.

44. Proposition 4. Given the plan and elevation of a point to pass through it a plane parallel to two lines whose plans and elevations are given.

Discussion. If through the given point two lines be drawn parallel to the two given lines, the plane of these two lines will be the required plane.
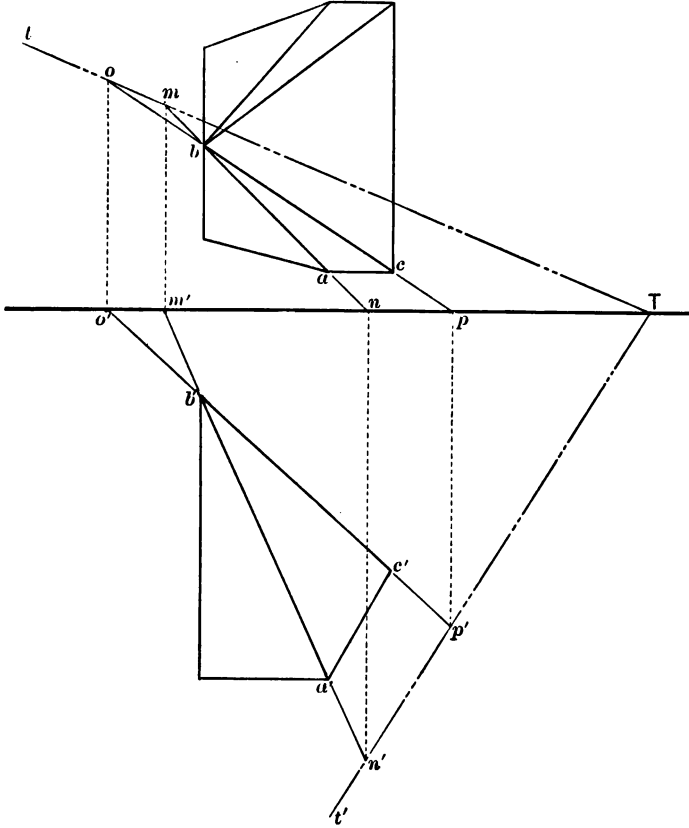
FIG. 46.

Construction. Let the problem be to cut off the trough, Fig. 47, one of whose edges is NP, by a plane through P parallel to MN and NO. Through p draw pq and pr parallel to the plan of the two given lines, mn and no. Also draw p'q' and p'r' parallel to m'n' and n'o'. Find now where these lines, PQ and PR, which are parallel to the given lines, pierce H and V. PQ pierces H at X and V at q'. PR pierces V and y'. Draw q'y' and extend it to T; also draw Tx. These are the H and V

traces respectively, and since PR is parallel to H the trace tT should be parallel to pr.   The plane T then is the required plane through P parallel to MN and MO.
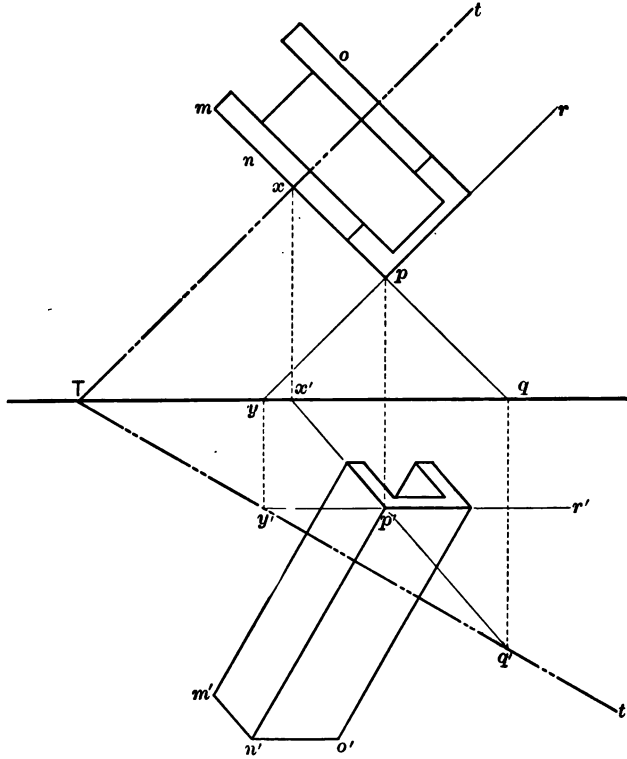
FIG. 47.

45. Special Cases.   Pass a plane through a point parallel to two given lines:

1. When one of the lines is parallel to the G. L.
2. When one of the lines is parallel to the H or V.
3. When one line is parallel to H and the other is parallel to V.
4. Pass a plane through one line parallel to another.

46. Proposition 5.   Given the plan and elevation of a point to pass through it a plane parallel to a given plane.

Discussion.   Since the required plane is to be parallel to the given plane the corresponding traces of these two planes will be

parallel. The direction of the required traces is, therefore, known. To find their location a line may be drawn through the given point parallel to either trace of the given plane and its H
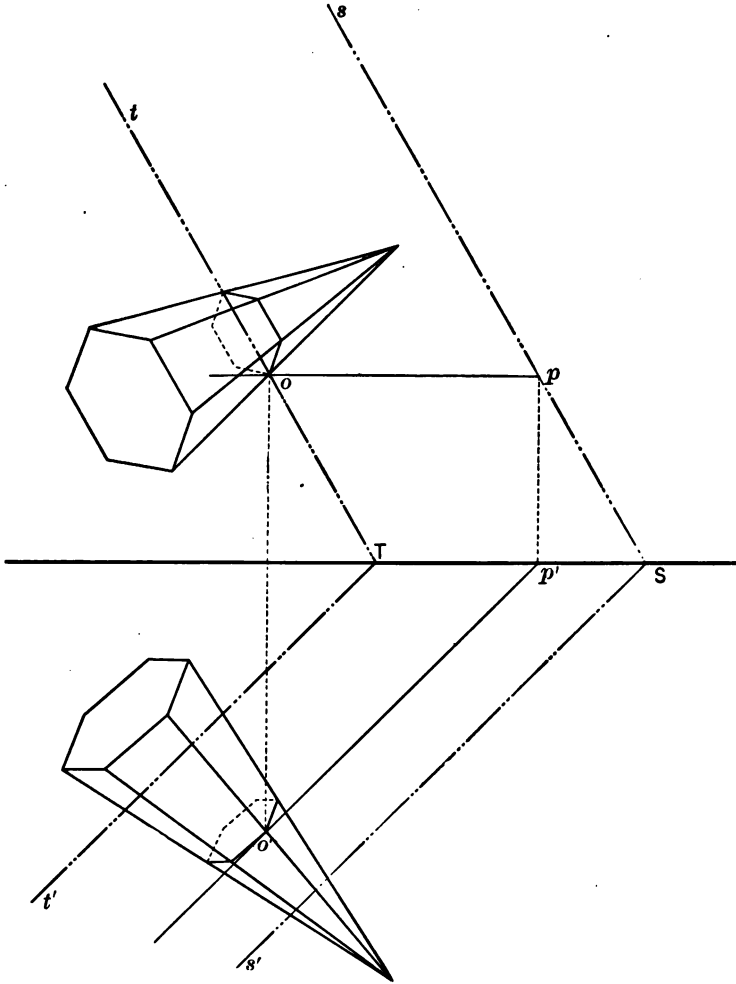
FIG. 48. ·

or V piercing point found. With one point on either trace and the direction of the traces known the plane may be located.

Construction. In Fig. 48 is shown the plan and elevation of an hexagonal pyramid whose base lies in the plane T. The prob-

lem is to cut off the prism by a plane parallel to T through the point O.

Through O draw a line OP parallel to t′T. This line pierces H at P. Through P draw sS parallel to tT and through S draw s′S parallel to t′T; sSs′ is the required plane through O parallel to the plane T.,
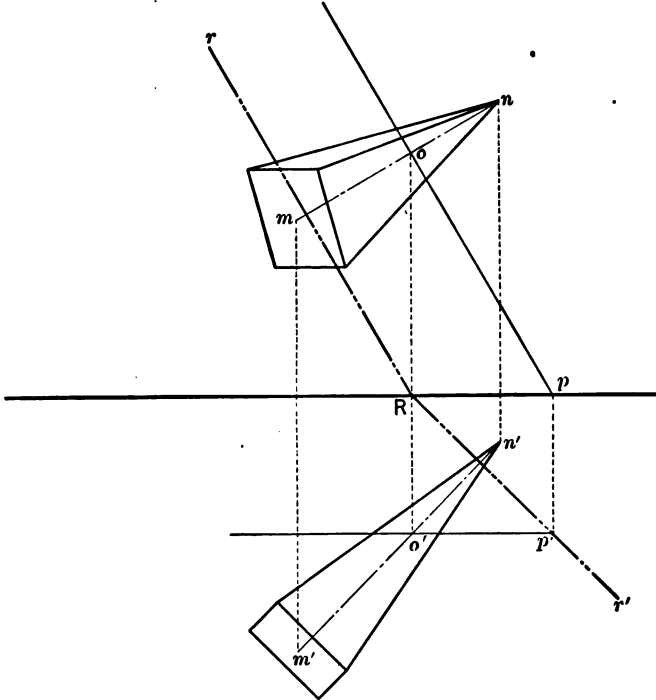
FIG. 49.

47. Special Case. Pass a plane through a point parallel to a plane which is parallel to the G. L.

48. Proposition 6. Given the plan and elevation of a line to draw through a point, whose plan and elevation are given, a plane perpendicular to the line.

Discussion. When a line is perpendicular to a plane the plan and elevation of the line are perpendicular to the corresponding traces of the plane. Thus the direction of the traces of the required plane is known. If then a line be drawn through the

given point parallel to the direction of either of these traces this line will be in the required plane. By finding its H or V piercing point the location of one point on one of the traces may be found and the plane located.

Construction. Let the problem be to find the traces of a plane through the point O which will cut a section perpendicular to MN, the axis of the pyramid in Fig. 49. The H trace of the required plane will be perpendicular to the plan view of the axis, and the V trace will be perpendicular to the elevation. Therefore, through O, the given point, draw a line parallel to the direction of the required H trace; its plan view will be op and its elevation o'p'. This line OP is a line of the required plane since it contains a point in the required plane and is parallel to one of the traces of that plane. It pierces V at p' and if through p' r'R be drawn perpendicular to m'n' the elevation of the axis, it will be the V trace of the required plane. The H trace is Rr which is perpendicular to the plan view of the axis. rRr', then, is the required plane.

49. Special Cases. 1. Construct this same problem when the point does not lie on the line to which the plane is to be perpendicular.

2. Find the location of a plane which is parallel to a given plane at a given distance from it.

PROBLEMS ON THE PLANE

108. The point A = 0; −2; −3; the point B = +1; −1; −1; the point C = +2; −3; −2. Find the traces of the plane of the triangle ABC.

109. The point A = +1; −1; −3; the point B = +1; −2; −1; the point M = +3; −1; −3; the point N = +3; −2; −1. Pass a plane through AB and MN and show its traces.

110. The line AB is parallel to the G. L. 1″ below H and 2″ back of V. The line MN is parallel to the G. L. 1″ above H and 2″ in front of V. Find the traces of the plane of these two lines.

111. The point M = −1; +2; +2; the point N = −2; −1; −1; the point O = −2; +2; −1; the point P = 0; −1; −2. Pass a plane through P parallel to the lines MN and ON.

112. The point A = 0; −1; 0; the point B = +2; −1; −3; the point C = +2; −3; −3; the point O = −2; −3; −1. Pass a plane through O parallel to AB and BC.

113. The plane T = ∞ ; −2; −4; the point O = −3; −2; −3. **Pass** a plane through O parallel to plane T.

114. The plane S = o; +6o; −6o; the point O = +2; −1; −3. **Pass** a plane through O parallel to plane S.

115. The point M = −1; −2; −1; the point N = −2; −3; −2. Pass a plane through M perpendicular to the line MN.

116. The point A = −1; −3; − 4; the point B = −3; −1; −1. **Pass** plane through AB perpendicular to it at its middle point.

117. The point M = o; o; o; the point N = −1; −3; −3; the point O = −3; −1; −2. Pass a plane through O perpendicular to MN.

CHAPTER IX

PROBLEMS ON ANGLES

50. Proposition 7. Given two views of an angle to find its true size.

Discussion. If the plane of the angle be revolved into co-incidence with H or V the angle in this position will be shown in its true size. Therefore, find the plane of the angle, and using either the H or the V trace as an axis revolve the angle into H or V. When the angle coincides with H or V it will be shown in its true size.

Construction. In Fig. 50 let the problem be to find the true size of all the angles between the edges of the section of the triangular prism; or, in other words, the shape and size of the triangle ABC. By Proposition 3 find the plane of the triangle; its traces are rR and r'R. With the V trace r'R, as an axis, revolve the triangle into V by the method in Article 36. When coinciding with V the triangle will occupy the position $A_V B_V C_{\bar{V}}$, and as this is its true shape and size the edges and the angles may be measured with scale and protractor.

51. Corollary. Given two views of one side of an angle, the traces of its plane, and its size, to construct the views of the other side.

Discussion. If the given side be revolved into H or V about the trace of the given plane, the angle may be constructed in this position in its true size. If now the plane be revolved back to its original position the two views of the required angle may be found.

Construction. Let the problem be, in Fig. 51, to construct a rectangular opening in the plane R with MN as one edge and the other edge one-half as long. By Article 34 revolve MN about rR into H. In this position MN will lie at $M_H N_H$ and the plane R will lie at rRr'_H. With $M_H N_H$ as one side construct the rec-

tangle $M_HN_HO_HP_H$ making M_HP_H equal to $\frac{1}{2}$ M_HN_H. This rectangle is the true shape and size of the required rectangular opening.

Now revolve rRr'_H back to its first position rRr' and with it revolve the rectangle $M_HN_HO_HP_H$. To find the plan of P draw
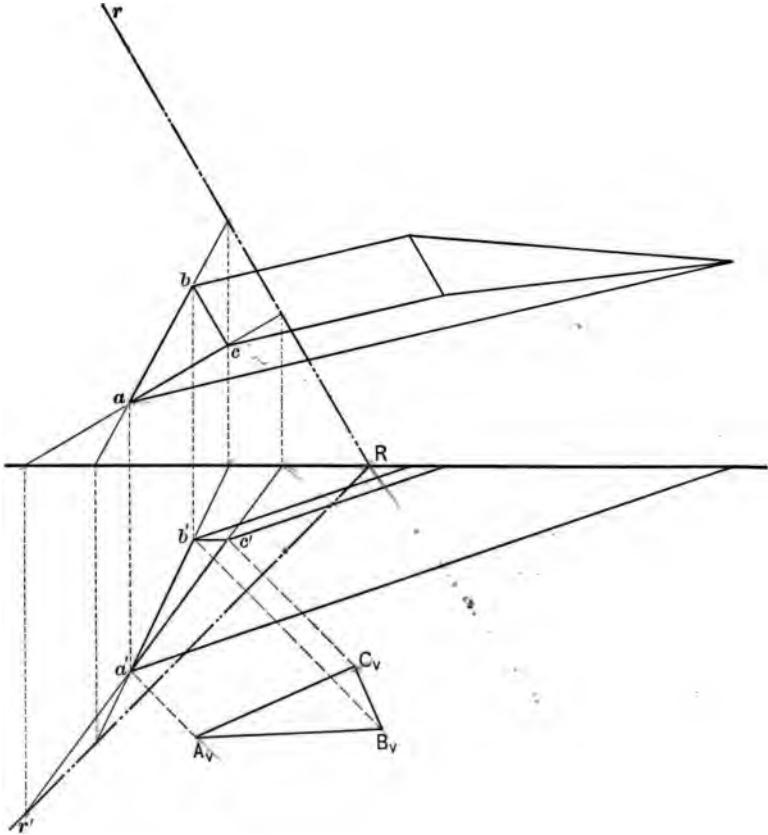
FIG. 50.

the diagonal N_HP_H and let it intersect the trace rR at a. Now when the plane R returns to its first position N_H revolves to n and the diagonal aB_H revolves to ab. Since P_H is on this diagonal p, the required plan view of P will be found at the point where a perpendicular from P_H to rR intersects ab. To find the elevation of P draw first $a'b'$, a' being on the G. L. since

A is on the trace rR, and b′ being on r′R. p′, the elevation of P, will be found on a′b′ in a perpendicular to the G. L. from p. In like fashion, O may be found and the plan and elevation of the required rectangular opening constructed.
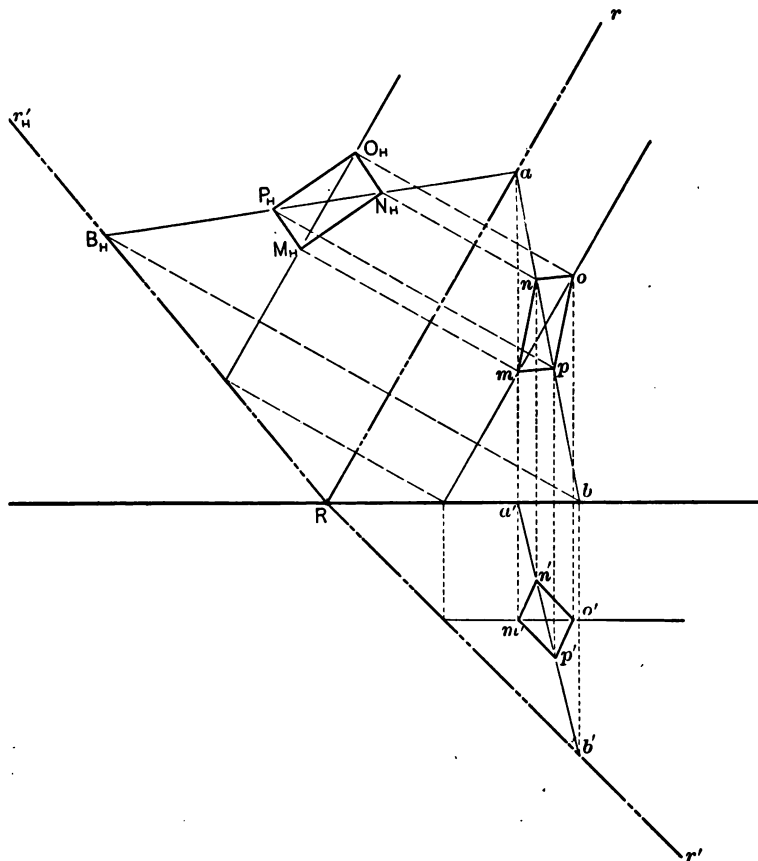
FIG. 51.

52. Corollary. Given a line and a point to draw through the point a line making a given angle with the given line.

Discussion. If a plane be passed through the given point and line it will be the plane of the required angle. If this plane be revolved into H or V the true relation of the point to the line will be shown when they are in the new position. The given angle may then be constructed. If now the plane be revolved

back to its original position the plan and elevation of the required angle may be found.

Construction. Let the construction be made according to the discussion.

53. Special Cases. 1. To find the size of an angle when one side is parallel to the G. L.

2. To draw two views of the bisector of an angle.

PROBLEMS ON ANGLES

118. Given the triangle ABC, A = 0; −3; −2; B = −1; −1; −1; C = −2; −2; −3, to find its true shape and size.

119. The point A = 0; 0; −3; the point B = +1; 0; −1; the point C = +2; 0; −2; the point O = −1; −3; −1. Find the true shape and size of each of the faces of the triangular pyramid whose base is ABC and upon whose apex is O.

120. The point M = 0; −3; −1; the point N = 0; −1; −3; the point O = −2; −1; −1. Find the true shape and size of the angle MNO.

121. The plane S = 0; −60; +45. The point O in this plane lies at +2; +1; +x. With O as one corner construct an equilateral triangular hole in the plane S with sides 1″ long. Let the base of the triangular hole be parallel to the H trace of the plane.

122. The plane T = −3; −45; +60. A line lies in this plane at M = −5; −x; −1 and N = −6; −2; y. With MN as one side construct a square hole in plane T.

123. The plane S = 0; +45; −60.   The point O = +2; −1; x. Through the point O, which lies in plane S, draw a line in plane S perpendicular to the H trace of S and find the angle it makes with the V trace.

124. M = −3; −2; −2.   N = 0; −1; −1.   O = −2; −1; −3. Draw a line through O making an angle of 30 degrees with MN. (Two solutions possible.)

125. M = −3; −1; −1.   N = 0; −2; −3.   O = −2; −4; −1. Through O draw a line perpendicular to MN.

126. A = −1; +2; +2.   B = −1; +3; +4.   O = 0; +1; +1. Through O draw a line perpendicular to AB.

127. A = 0; +1; +4.   B = 0; −1; −2.   C = +2; −3; −1. Show three views of the bisector of the angle ABC.

128. An observer stands in a lighthouse tower at A = +1000′; +300′; +100′, and at 10 o'clock observes a ship N. 30° E. from him at an angle of depression of 30 degrees. One hour later the same ship is N. 60° W. from him at an angle of depression of 15 degrees. Assuming the ship keeps the same course and speed how will it bear from him and what will be its angle of depression at 12 o'clock?

CHAPTER X

PROBLEMS ON POINTS, LINES, AND PLANES

54. Proposition 8. To draw the plan and elevation of the intersection of two given planes.

Discussion. Since the line of intersection contains all of the points common to both planes, its location will be determined if any two of these common points be located. If, then, a line
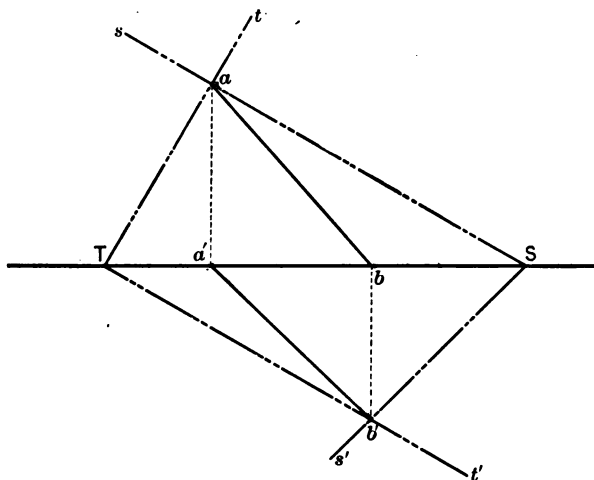
FIG. 52.

be drawn from the intersection of the H traces to the intersection of the V traces, the plan and elevation of this line show the required intersection.

Construction. Let T and S, Fig. 52, be the given planes whose intersection is required. The H traces intersect at a point whose plan is a and whose elevation is a′. The V traces intersect at a point whose plan is b and whose elevation is b′. ab, then, is the plan and a′b′ is the elevation of the required intersection AB.

63

In case the traces of the given planes do not intersect conveniently on the drawing, Fig. 53 shows a method of finding the required intersection without using the intersection of the traces.   T and S are again the given planes, Rr is an auxiliary plane parallel to V, which cuts the line AB from the plane S, and the line BC from the plane T, thus giving a point B common to both planes.   In like manner a second point N on the required
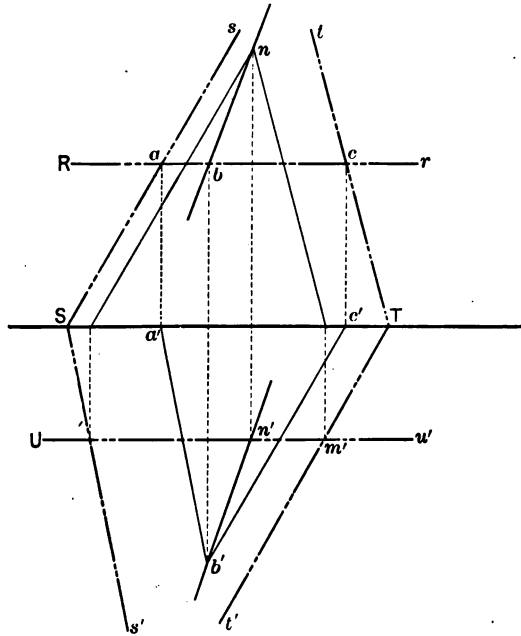
FIG. 53.

intersection may be found by passing an auxiliary plane U through S and T parallel to H.   B and N when joined give the plan view bn and the elevation b'n' of the required intersection of the planes S and T.

55. Special Cases. 1. Find the intersection of two planes when one pair of traces are parallel.

2. Find the intersection of two planes which are parallel to the G. L.

56. Proposition 9. Given a plan and elevation of a line to find where it pierces a given plane.

· **Discussion.** If any plane be passed through the given line it will contain all points of that line. It will, therefore, contain the required point. Since the required point lies in the given plane and in the auxiliary plane it will lie on their line of intersection. If then the line of intersection be found and the point
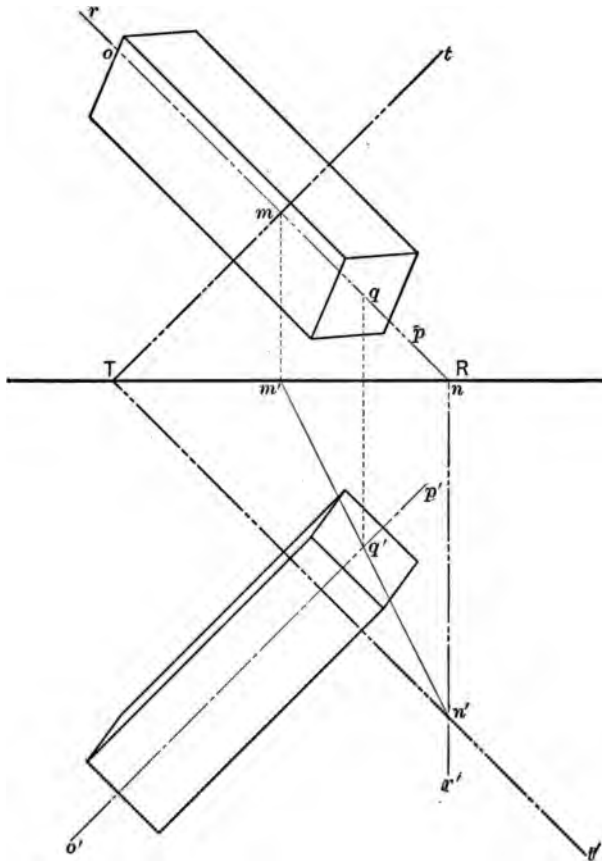
Fig. 54.

common to it and the given line be located it will be the required piercing point.

Construction. In Fig. 54 is shown the plan and elevation of a timber whose center line is OP. The problem is to find the point where this line OP pierces the plane tTt'. Pass the auxil-

iary plane R through OP. Since the required piercing point lies
in the plane R and in the given plane T it will lie on their inter-
section MN. (See Proposition 8.) The required point lies on
MN and OP; therefore, it will be at the intersection of these two
lines or at Q. q then is the plan and q' is the elevation of the
required point.

It should be noted that any auxiliary plane through OP will
serve, but, for convenience in drawing, one which is perpendicular
to H or V should be used. The points where the edges of the
timber pierce T may be found in a similar manner and when
joined give the plan and elevation of the section cut from the
timber by plane T.

57. Special Cases. Find where a line pierces a plane:
1. When the line is parallel to the G. L.
2. When the plane is parallel to the G. L.
3. When the plane is parallel to the G. L. and the line lies in
   a profile plane.
4. When the plan and elevation of the line are parallel respec-
   tively to the H and V traces of the given plane.

58. Proposition 10. Given the plan and elevation of a point
to find its shortest distance from a given plane.

Discussion. The shortest distance from a point to a plane is
the perpendicular distance from the point to the plane. If the
perpendicular be drawn from the point to the given plane and
the point where the perpendicular pierces the plane be found,
the distance between these two points will be the required
shortest distance.

Construction. In Fig. 55 the problem is to find the altitude
of the pyramid whose base lies in the plane T and whose apex is
at O. From O draw a line perpendicular to the plane T. (See
Observation f, Article 23.) Find where this line pierces plane T
by Article 56. p is the plan and p' the elevation of this piercing
point. Then op is the plan and o'p' is the elevation of the
shortest distance from the apex of the pyramid to the plane of
its base. Its true length may be found by Article 38, as shown
at o''p'.

It should be noted that P, which is the foot of the perpendicu-

lar drawn from O to the plane T, is the projection of the point O on the oblique plane T.
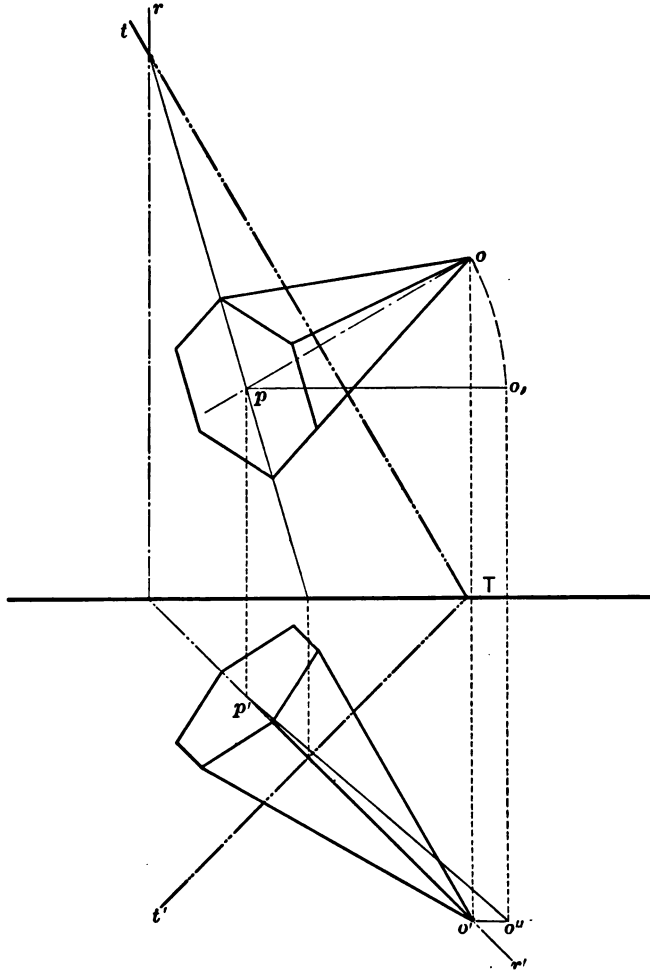
FIG. 55.

59. Corollary. Given a plane, the plan and elevation of the projection of a point upon this plane, and its distance from the plane to find the plan and elevation of the point.

Discussion. If a perpendicular be erected to the given plane through this given projection of the point, the required point

will lie in this line.   To find its location a point on the perpendicular must be found at the given distance from the plane.

Construction.   Let the construction be made in accordance with the suggestions in the discussion.

60. Special Cases.   1. Find the shortest distance from a point to a plane when the plane is parallel to the G. L.

2. Given two parallel planes to find the shortest distance between them.

PROBLEMS

129. The plane S = o; −45; +6o.   The plane T = +3; −300; +300. Find the true length of that portion of the intersection of planes S and T which lies between H and V.

130. The plane S = ∞ ; −3; −1.   The plane T = ∞ ; −1; −3.   Find the plan and elevation of the intersection of these planes.

131. The plane S = o; +6o; −6o.   The plane T = ∞ ; −4; −3.   Find the plan and elevation of the intersection of planes S and T.

132. The plane S = o; −90; −75.   The plane T = −2; −75; −6o. Find the plane and elevation of the intersections of S and T.

133. The plane T = −2; −90; −300.   The plane S = +1; −90; +300.   Find the plan and elevation of the intersection of planes S and T.

134. The plane S = −2; +45; −6o.   The point M = −2; −1; −5. The point N = +2; −4; −1.   Find where the line MN pierces the plane S.

135. The plane S = ∞ ; −4; −2.   The point M = −3; −1; −4. The point N = o; −3; −1.   Find where MN pierces plane S.

136. The plane S = o; +75°; −30°.   The line MN is in the third quadrant parallel to the G. L. 2″ from H and V.   Find where the line MN pierces plane S.

137. The point M = o; −1; −2.   The point N = −3; −1; −4.   The plane S = −3; −60°; −6o°.   Find where MN pierces plane S.

138. The plane S = o; +135°; −105°.   The point O = −1; −3; −2. Find the shortest distance from O to plane S.

139. The plane R = ∞ ; −3; −2.   The point O = −o; −3; −4. Find the shortest distance from O to the plane R and show its plan and elevation.

140. The plane S = o; +6o°; −45°.   Pass a plane parallel to plane S and 2″ from it.

141. The plane S = o; +75°; −6o°.   The plane R = −2; +75°; −6o°. Find the shortest distance between the planes.

142. The plane S = o; −45°; +6o°.   The point M = −2″; −2″; x. M is one corner of a regular square pyramid whose base lies in plane S.   The base of the pyramid is a 2″ square and the altitude is 3″.   Draw a plan,

elevation, and end view of the pyramid when one edge of its base is parallel to sS.

143. The plane R = o; +45°; −60°. The point O is +3″; −y″; −2″. This point is the center of a 1″ square which lies in plane R with one edge making an angle of 30 degrees with Rr. Draw the plan, elevation, and left end view of a cube, one of whose faces is this square.

144. A point of light is located at +3; +6; +3. A ray of light from this point strikes a mirror in H at the point o; +2; o and is reflected to V. Find the coördinates of the point where the reflected ray strikes V. (It will be remembered that the angles of incidence and reflection are equal.)

145. A ray of light comes from the point +6; +5; +4, and strikes a mirror at the point +3; +3; +2. The plane of the mirror inclines 60 degrees to V and is perpendicular to H. Find where the reflected ray strikes V.

61. Proposition 11. Given the plan and elevation of a line to find the plan and elevation of its projection upon any given plane.

Discussion. The projection of any point upon any plane is the foot of a perpendicular from the point of the plane. ·If perpendiculars be drawn from any two points of the given line to the given auxiliary plane and the points where these perpendiculars pierce this plane be found, these piercing points when joined will be the required projection of the line.

Construction. The problem in Fig. 56 is to find the plan and elevation of the projection of the face MNOP of the casting on the plane R to which this face is parallel. From each corner of the face draw a perpendicular to the plane R and find where these perpendiculars pierce the plane. The plan view of the projection on R is abcd — these points being the plan view of the piercing points of the perpendiculars — and the elevation is a′b′c′d′. $A_H B_H C_H D_H$ shows the face in its true size after it has been revolved into H about rR as an axis, and set to one side of the actual plane of revolution so as to avoid confusion in the drawing.

62. Special Cases. To project a line upon any plane:
1. When the line is parallel to the G. L.
2. When the plane is parallel to the G. L.
3. When the plan and elevation of the given line are parallel to the corresponding traces of the given plane.

63. Proposition 12. Given the plan and elevation of a line and of a point, to find the shortest distance between them.

Discussion: First Method. If the line and the point be revolved into H or V about the trace of the plane which contains
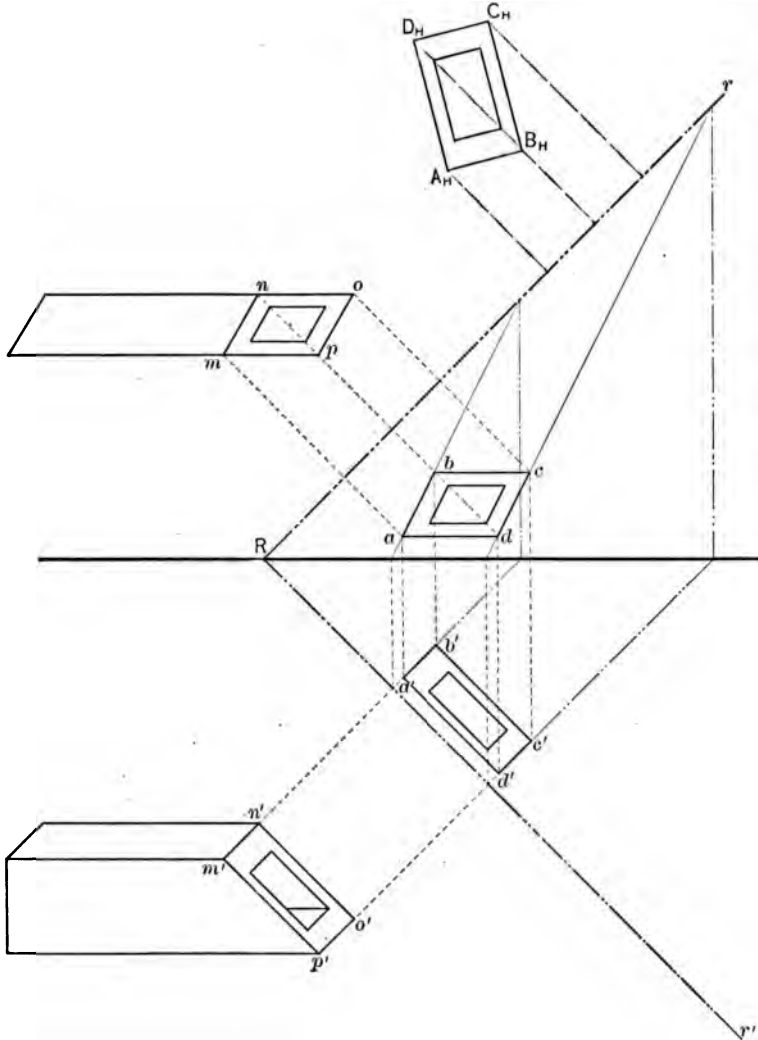
Fig. 56.

them they will be shown in this revolved position in their true relation. If now from the revolved position of the given point a perpendicular be drawn to the revolved position of the line,

this perpendicular will be the measure of the shortest distance from the point to the given line. If a plan and elevation of this distance be required the point must be revolved back to the first position, and with it the perpendicular.

Construction. Let the construction be made in accordance with the discussion.

Discussion: Second Method. It not infrequently happens that the relation of the given point to the given line is such as to make the construction by the first method awkward. In such cases it will be found more convenient to pass a plane through the point *perpendicular* to the line, and by finding where the given line pierces this plane a point will be found which, when joined to the given point, will be the shortest distance from the given point to the given line.

Construction. Fig. 57 shows such a case. The problem is to find the length of the center line required for a timber to reach from point A to the second timber whose center line is OQ. Through A the plane R is passed perpendicular to OQ. (Article 48.) The line OQ pierces this plane at B. (Article 56.) AB then is the required shortest distance and ab_1 shows its true length.

64. Proposition 13. Given the plans and elevations of two lines not in the same plane to find the plan and elevation of the shortest line which can be drawn between them.

Discussion. If through one of the given lines a plane be passed parallel to the other, and if upon this plane the second line be projected, this projection will intersect the first line at a point which will be one end of the shortest connecting line. If now at this point a line be erected perpendicular to the plane it will intersect the second line at a point which will be the other end of the shortest connecting line. If the length of this line be required it may be found as in Article 38.

Construction. Fig. 58 shows two timbers lying in different planes. The problem is to find the shortest third timber to join them — center lines only being considered. Through PQ pass a plane parallel to AB. This plane is rRr'. Upon plane R project AB. (Article 61.) This projection on R is shown by
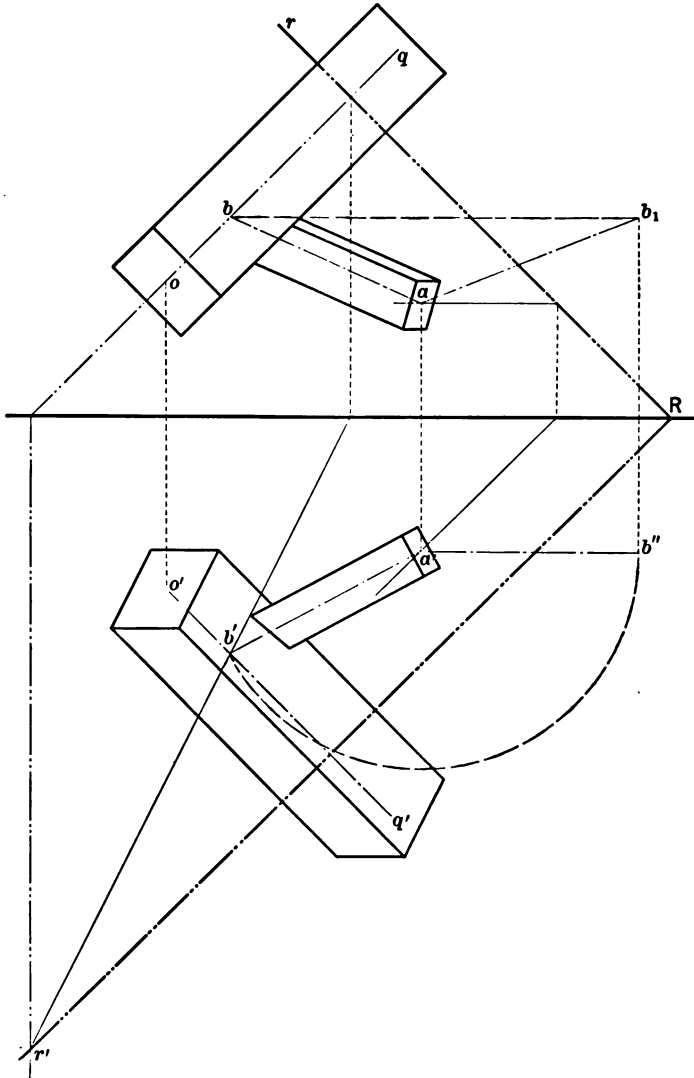
FIG. 57.

mn and m'n'.   Now at O, where PQ and MN intersect, erect a perpendicular to the plane R.   This perpendicular intersects AB at C.   CO then is the center line of the shortest timber that can connect PQ and AB.   Its true length may be found as in Article 38.
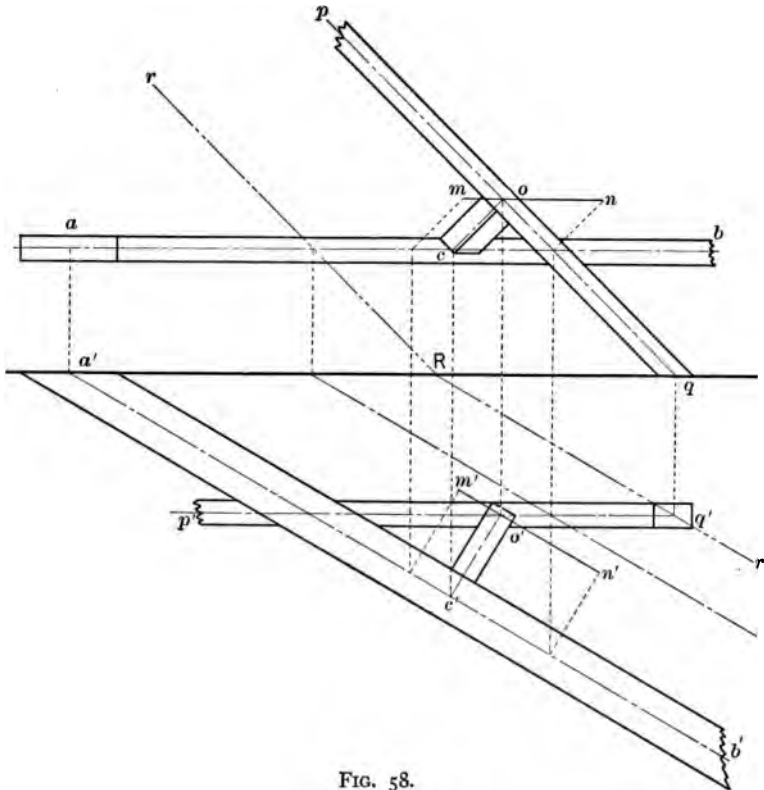
Fig. 58.

65. Proposition 14.   Given the plan and elevation of a line to find the true size of the angle it makes with a given plane.

Discussion.   The angle a line makes with a plane is equal to the angle a line makes with its projection on that plane.   If, therefore, the given line be projected on the given plane and the true size of the angle between the given line and the projection on the given plane be measured it will be the required angle the given line makes with the given plane.

Construction. In Fig. 59 the problem is to find the angle between the plane S and the center line NQ of a tunnel. By
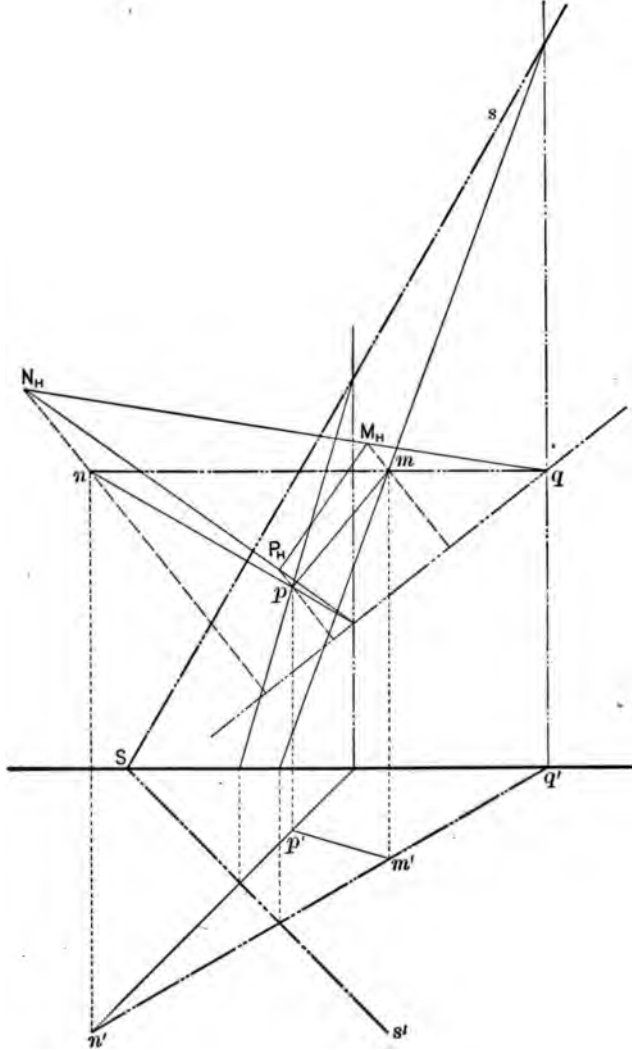
FIG. 59.

Article 56, find where the center line NQ pierces the plane S. This point is M, and it is the vertex of the required angle. From any other point N of NQ draw a line perpendicular to the plane

S and find where it pierces the plane S. This point is P, shown in plan at p and in elevation at p′, and MP is the projection of MN upon S. NMP, then, is the angle the line NQ makes with the plane S. Its true size is shown at $N_H M_H P_H$. (Article 50.)

Further Discussion. In case only the size of the angle the line makes with the plane is required a shorter method may be used.
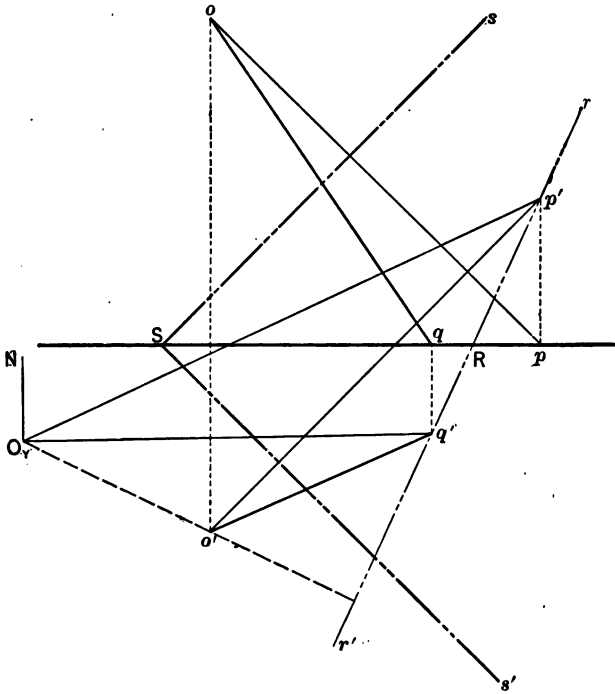
FIG. 60.

In the problem just solved it will be observed that since the angle MPN is a right angle — NP being perpendicular to MP — the angle PNM is the complement of the required angle. If, therefore, from any point of the given line a perpendicular be drawn to the given plane the angle between the perpendicular and the given line will be the complement of the required angle. The true size of this complement may be found and subtracted from 90 degrees thus giving the size of the required angle.

Construction. Fig. 60 shows the method of finding the size of the angle between the plane S and the line OQ. The perpendicular from O to S pierces V at P; OQ pierces V at Q. r'R is the V trace of the plane of the angle QOP and its true size may be found as shown. Subtracting q'O$_v$p' from 90 degrees gives NO$_v$p', the size of the angle OQ makes with S.

66. Special Cases. 1. Find the angle a given line makes with H, V, or P.

2. Given the plan and elevation of a line and of its projection on some plane to locate the traces of the plane.

PROBLEMS

146. The plane T = o; +60; −60. The point M = o; −1; −1½. The point N = +2; −1; −1. Find the plan and elevation of the line MN when projected upon plane T.

147. The plane S = ∞; −3″; +4″. The line MN is parallel to the G. L. 1″ from H and V in the third quadrant. Find the plan and elevation of MN when projected upon plane S.

148. The point M = o; o; o. The point N = +3; −3; −3. The point O = +2; −3; −1. Find the shortest distance from O to MN and show its plan and elevation.

149. A tunnel runs from a point N = o; −6; o S. 60° W. on a falling 6 per cent grade. The bottom of a shaft is located at a point −3; −2; −10. Find the actual length of the shortest tunnel which will connect the bottom of the shaft and the tunnel and show its plan and elevation.

150. A drain runs N. 30° E. from a point o; o; −1 on a falling 10 per cent grade. Show the plan and elevation and find the length of the shortest connection between the drain and the point +1; −2; o.

151. A steam pipe runs through the points M = o; −3; −1 and N = +3; −6; −1. A second pipe runs through the points A = −1; −1; −4 and B = +2; −1; −2. Find the shortest connection between the pipes and give its true length. Use center lines only.

152. A tunnel joins the bottoms of two shafts which are located at o; −8; −50 and +50; −25; −75. A second tunnel starts at o; o; −75 and runs N. 45° E. on a rising 20 per cent grade. Draw a plan and elevation of the shortest connecting tunnel which can join these two and give its actual length.

153. The plane of an ore body is located at o; +75°; −60°. This plane is cut by a tunnel at the point O = +2; −3; x. The tunnel runs S. 60° E. through this point on a rising 10 per cent grade. Find the angle the tunnel makes with the plane of the ore.

154. The plane of a body of ore is located at S = o; +75°; −60°. A shaft is sunk at a point +3; −3; o. Find the angle the shaft will make with the plane of the ore and how deep the shaft will have to be in order to reach the ore.

155. A brace 14′ long meets the ground 8′ in front of a wall and rests against the wall 6′ above the ground. Find the angles the brace makes with the ground and the wall.

67. Proposition 15. Given the angles a line makes with H and V, and the plan and elevation of some point on the line, to draw the plan and elevation of the line.

Discussion. If a line of any assumed length be drawn through the point parallel to H and making the required angle with V the plan view of the line will be equal in length to the plan view of the required line. Also, if a line be drawn through the given point parallel to V and making the required angle with H the elevation of this line will be equal in length to the elevation of the required line. Having the length of the two views and the location of one point the line may be constructed.

Construction. In Fig. 61, O is the bottom of a shaft from which runs a tunnel with a given grade and direction. The grade is equivalent to the angle it makes with H and its direction is equivalent to the angle it makes with V.

Draw a line through O parallel to H making the given direction with V. Assume OQ as the length of the tunnel. o′q″, then, is the length of the elevation of the required line. Likewise draw through O a line parallel to V and making the given grade with H, and make it equal in length to OQ. op,, then, is the length of the plan view required. Revolve these two lines until P and Q coincide at N; on and o′n′ then will be the required plan and elevation of the center line of the tunnel.

NOTE. The sum of the angles given must not exceed 90 degrees or the problem becomes impossible.

68. Proposition 16. Given two planes to find the angle between them.

Discussion. The angle between the planes is measured in a plane perpendicular to both planes or to their line of intersection. If, then, a plane be passed through any point of a line common

to both planes and the intersection of this plane with both of the
others be found the angle thus formed will be the required angle
between the given planes, and its size may be determined.

Construction. In Fig. 62 is shown a plan and elevation of a
trough. The problem is to find the angle between its sides.
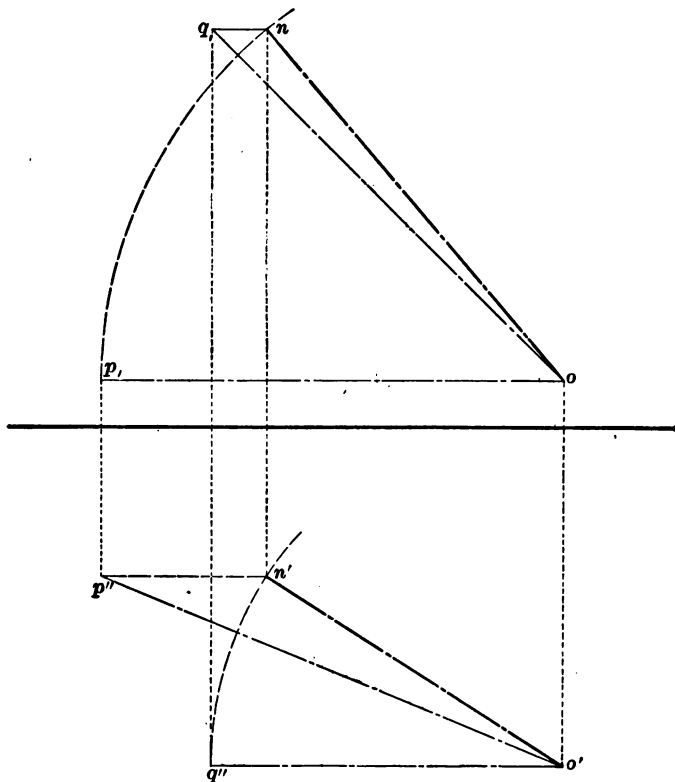
FIG. 61.

Assume the point O on the line common to the planes of the two
sides, and through this point, by Article 48, pass a plane perpen-
dicular to the line. This plane is rRr'. Now by Article 54 find
the intersection of R and S, and R and T. This is OP in the first
case and OQ in the second, and POQ is the required angle. Its
true size, pO_Hq, may be found by Article 50.

69. Special Cases. 1. Find the angle a plane makes with H
and V.

2. Find the angle between two planes which are parallel to the G. L.

70. Proposition 17. Given one trace of a plane and the angle the plane makes with the corresponding plane of projection to find the other trace.
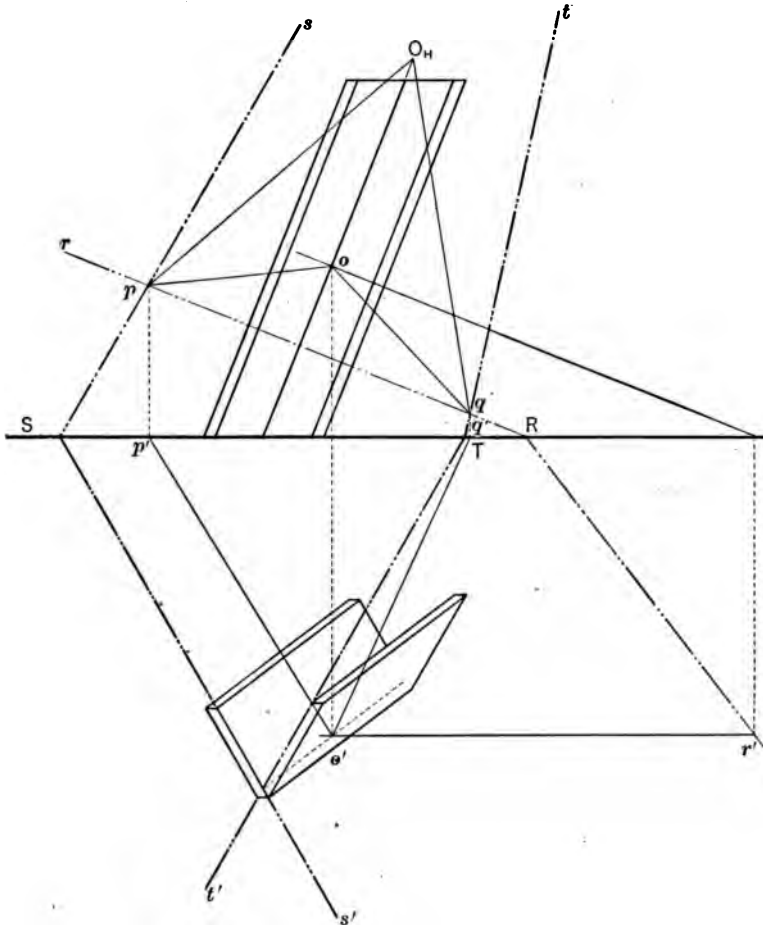
FIG. 62.

Discussion. The vertex of the given angle will lie on the given trace and the angle will lie in a plane perpendicular to the given trace. This angle may, therefore, be constructed and since one

side of it lies in the required plane the piercing point of this side
will lie on the required trace.

Construction. Let the problem be, Fig. 63, to find the plane
of the ore body whose strike is the line sS (the "strike" is the
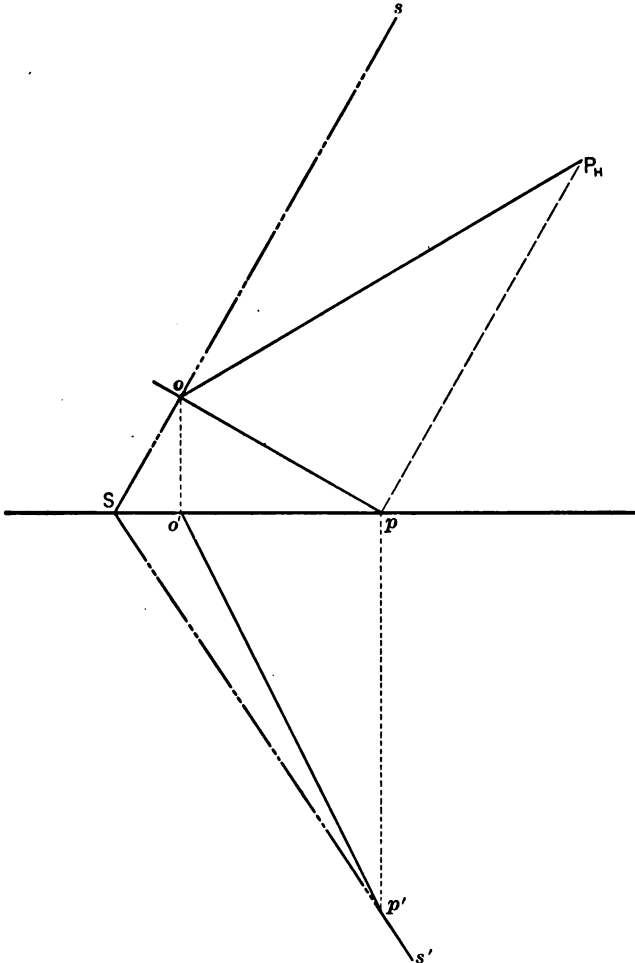
FIG. 63.

direction of any horizontal line in the plane) and whose dip is
60 degrees (the "dip" is the inclination of the plane to the hori-
zontal). Lay off an angle of 60 degrees with its vertex o on sS

and one side OP perpendicular to sS. $P_H$op will be the revolved position of the given angle the ore body makes with H, and P_H will be the revolved position of the point where OP pierces V. Since the line OP must lie in the plane of the ore revolve the angle till it is perpendicular to H, and P_H will fall at a point shown in plan and in elevation at p and p'.   P, then, is a point in s'S, the required trace.

FIG. 64.

71. Special Case.   Given the angles a plane makes with H and V and the location of one point in the plane to find the traces of the plane.

Discussion.   If through the given point a line be drawn perpendicular to the required plane this line will make with H and V angles equal to the complements of the angles the plane makes with H and V.   If, therefore, a line be drawn through the given point making with H and V complements of the given angles, this line will be perpendicular to the required plane, and the plane may then be located.

NOTE. The sum of the given angles must not be less than 90 degrees else the problem will be impossible.

Construction. Let O be the given point in Fig. 64 and let it be required to find the traces of the plane through O which makes 60 degrees with H and 45 degrees with V.

By Article 67, draw a line through O making an angle of 30 degrees with H and 45 degrees with V. Now, by Article 48, pass through O a plane perpendicular to the line. sSs′ is such a plane and since it is perpendicular to a line making 30 degrees with H and 45 degrees with V the plane itself makes 60 degrees with H and 45 degrees with V. S, then, is the required plane.

<div align="center">PROBLEMS</div>

156. Through the point −6; −4; −3 draw a line 3″ long which will make angles of 30 degrees with H and V.

157. The bottom of a shaft is located at 0; −6; −10. Through this point a tunnel is driven making an angle of 60 degrees with V and 15 degrees with H. Give the bearing of the tunnel.

158. A guy wire is anchored at 0; −3; 0 and is fastened to the top of a stack at +3; +1; x. Find how high the stack is when the wire makes an angle of 60 degrees with the ground. Find also the angle the wire makes with V.

159. Find the angle between the end and side of a floor hopper; size of opening in floor 6′ by 8′, size of outlet 12″ by 12″, depth of hopper 3′.

160. Find the angles between the sides of a hood which fits in the corner of a room. The opening of the hood is 24″ square and its outlet is 6″ square. The distance between opening and outlet is 18″.

161. Three shafts are sunk to the plane of an ore body. One is located at the point −3; −4; −10; a second is located at 0; −1; −18; the third is located at +1; −6; −24. Find the angle the plane of the ore makes with H and V.

162. A V-shaped flume runs N. 30° E. on a falling 10 per cent grade. The bottom of the flume runs through the point −2; 0; −2 and the angle between the sides of the flume is 30 degrees. Draw a plan and elevation of the flume assuming it to be made up of a 2″ by 10″ piece fastened to a 2″ by 8″ piece.

163. An ore body dips 60 degrees and strikes S. 15° W. from the point −2; −6; −10. Show its outcrop and find the angle it makes with the vertical plane.

164. The bottom of a well is at 0; −6; −18. The plane of a stratum of clay through this point dips 60 degrees and makes an angle of 75 degrees with V. Find the traces of this plane.

165. The outcrop of a stratum of stone runs N. 60° W. from the point +6; −4; o and dips 15 degrees. Find the angle the stratum makes with V.

166. The wedge-shaped prow of a snow plough makes angles of 60 degrees with H and V, the planes of the faces inclining so as to meet. Find the angle at which a plate will have to be bent in order to form a shoe over the intersection of these planes.

CHAPTER XI

SURFACES

72. A *surface* is the area made by a line moving according to some law.

In descriptive geometry surfaces are classified according to the character of the moving line, called the *generatrix*, and the law controlling its motion.

73. The generatrix may be either a straight or a curved line and its motion may be controlled in one of several ways. It may be made to move so as to touch other lines either straight or curved, called *directrices*, and to remain parallel to a plane, called a *plane director;* or it may be made to revolve about another line, called the *axis of revolution;* or its motion may be controlled in other ways. All surfaces, however, are generated by a line whose motion is controlled in some specified way.

74. Surfaces thus generated are divided into two general classes: *ruled surfaces* and *double curved surfaces*. A *ruled surface* is generated by the motion of a straight line. A *double curved surface* is generated by the motion of a curved line. It is obvious, therefore, that a straight edge may be made to coincide with a ruled surface while it will coincide with a double curved surface in one point only; this fact offers a convenient method of identifying surfaces.

75. There are three kinds of ruled surfaces: *Plane, single curved*, and *warped*.

A *plane surface* is generated by a line moving so as to constantly touch straight line directrices which either intersect or are parallel. Thus, in Fig. 65, the line MN is the generatrix, and the traces of the plane, sS and s'S, are the intersecting directrices. The line MN, then, moves so as to constantly touch these two lines, thus generating the plane sSs'. In further discussions of plane surfaces it will be understood that surfaces

with plane faces are meant; the intersections of the plane faces, or planes, form the edges of the surfaces which hereafter will be called plane surfaces.

Any combination of intersecting planes, usually three or more, forms a plane surface. Many of these combinations have geometrical names such as *prisms*, *pyramids*, etc., with any number of faces from three upward. Some of these surfaces have commercial names which identify the uses to which they are put, as, for example, the hopper which geometrically is a
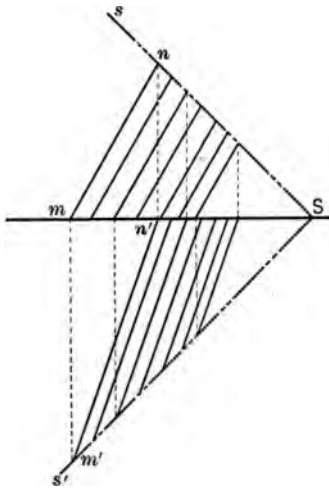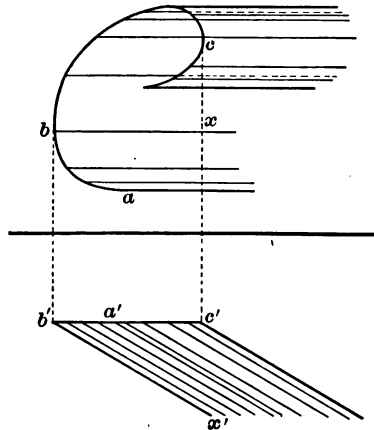
FIG. 65. FIG. 66.

truncated rectangular pyramid. It is with such surfaces as have commercial uses that this text has to do rather than with those which possess mathematical interest only.

76. There are three kinds of single curved surfaces: *cylindrical*, *conical*, and *convolute*. All of these surfaces are ruled and the rulings — or the positions of the generating line — are called *elements*.

A *cylindrical surface* is generated by a straight line moving so as to always remain parallel to its first position and to constantly touch a plane curve.[1] When this plane curve is closed the surface is called a *cylinder*. In Fig. 66 the generating line

[1] A plane curve is one which lies in a plane, as a circle; a space curve is one which does not lie in a plane, as a helix.

BX moves so that it remains parallel to its first position, BX, and constantly touches the curve ABC, thus generating a cylindrical surface.   In Fig. 67 the directrix is taken as a circle in H and since this is a closed curve the resulting surface is called a cylinder.   It will be noted that the elements of a cylinder are parallel to each other.   A *conical surface* is a surface generated by a straight line so moving that one point remains stationary while the line constantly moves along a plane curve.   This stationary point is called the *apex*, and it is obvious that since the generating line extends beyond the apex there will be generated two surfaces, called *nappes*, one on each side of the
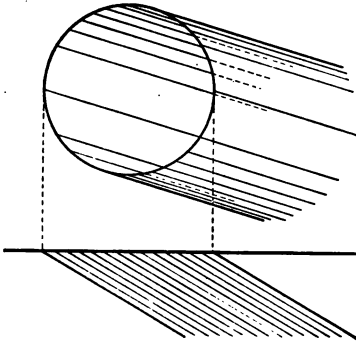
FIG. 67.
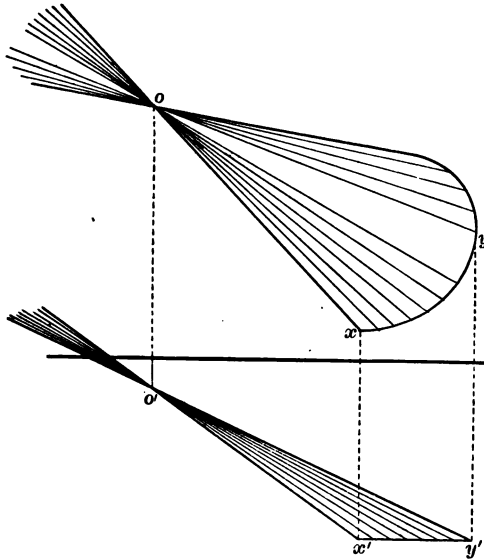
FIG. 68.

apex.   In discussions of conical surfaces in this text only one nappe will be considered, however.   When the plane curve

directrix is a closed curve the surface is called a cone. In Fig. 68 the generatrix moves so that it goes through the point O, the apex, and constantly touches the curve XY, thus generating a conical surface. In Fig. 69 the curved directrix is a circle and the resulting surface is, therefore, a cone.
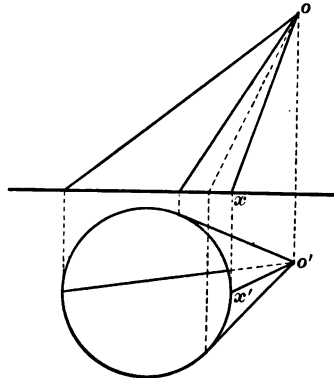
A *convolute surface* is generated by a straight line moving so as to be constantly tangent to a space curve. In Fig. 70 the space curve 1–2–3——11 is the curvilinear directrix to which the elements are constantly tangent, and a portion of the surface is shown by the positions of the generating line at 1, 2A, 3B,——7F. The

FIG. 69.
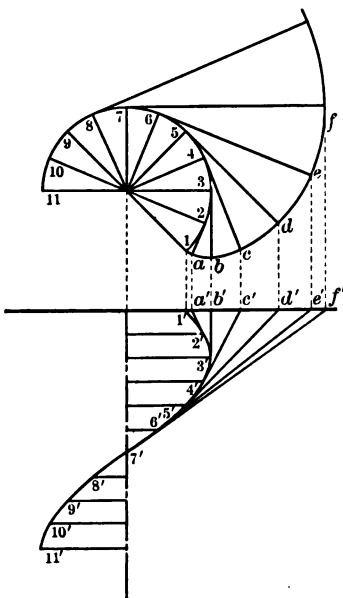
most important convolute surface commercially is the helical convolute, so-called because its curvilinear directrix is a helix, and since this surface is a special case of helicoid it will be considered in detail in connection with that surface.

77. A *warped surface* is a ruled surface generated by a straight line so moving that the elements of the resulting surface neither intersect nor are parallel. Thus in Fig. 71 the generating line AX so moves that the elements of the resulting surface neither intersect nor are parallel. In this surface the lines AB and CD are rectilinear directrices and the generatrix, AX, constantly touches them; the H plane is the plane director as all of the positions of AX remain parallel to this plane.

FIG. 70.

There are innumerable warped surfaces since there are innumerable combinations of generatrices, directrices, and plane directors but comparatively few of them are important commercially, and these will be discussed in detail in a later chapter.
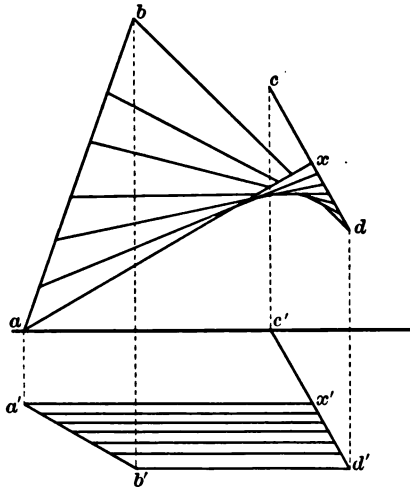
FIG. 71.

78. A *surface of revolution* is generated by the motion of a line about another line called the *axis of revolution*. The generating line may be either straight or curved; in case the generatrix is a straight line the resulting surface will be a *ruled* surface of revolution; if the generatrix is a curved line the resulting surface will be a *double curved* surface of revolution. In Fig. 72 the circle whose center is at O is revolved about the axis of revolution AB. The resulting surface is a double curved surface of revolution called an *annular torus*. In the figure one-quarter of the surface has been cut away in order to show the curvilinear generatrix. Whether the generatrix is straight or curved all surfaces of revolution have a circular section in planes perpendicular to the axis of revolution.
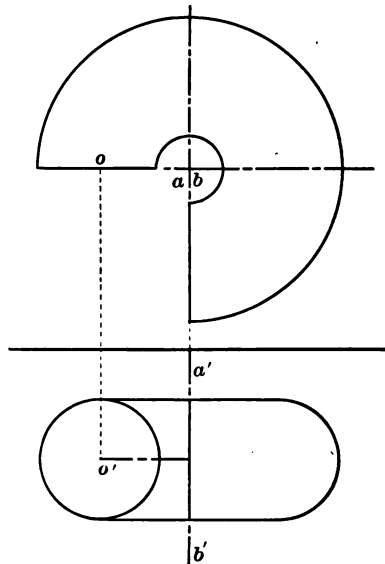
FIG. 72.

79. *Developable surfaces* are those which may be rolled out

on a plane surface so that all the elements will coincide with the
plane.   In Fig. 73 is shown a cone lying in a plane so that the
element OX coincides with plane.   If the cone be now rolled
out the curve of the base will rectify itself on the line *xx* as the
elements of the surface successively come in contact with the
plane.   The result, *oxx*, is called a *development* of the cone.
The only developable surfaces are the single curved surfaces as
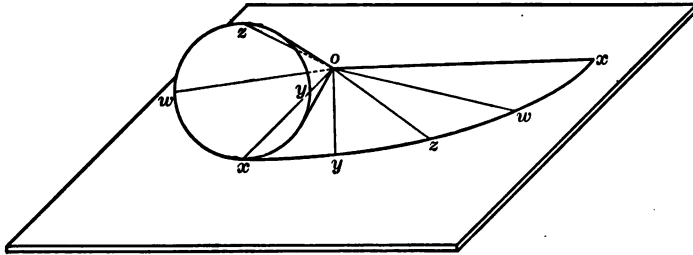
FIG. 73.

these obviously are the only surfaces whose elements may be
made to lie in a plane without changing their relation to each
other.   In practice, however, a number of warped and double
curved surfaces are considered developable; this is made possible
by approximate methods of development which give patterns
which conform closely enough to the original surface for practical
purposes.   The bottoms of hemispherical water tanks are good
examples of this fact.

CLASSIFICATION OF SURFACES

Surfaces are either

1. Ruled. Ruled surfaces may be

1. Plane surfaces such as { Prisms, Pyramids, Polyhedrons }

2. Single curved surfaces such as { Cylinders, Cones, Convolutes }

3. Warped surfaces such as { Cylindroids, Conoids, Helicoids, etc. } See page 155 for a complete classification

or

2. Double Curved. Double curved surfaces may be

1. Surfaces of Revolution such as { Spheres, Ellipsoids, Paraboloids, Hyperboloids, Tori } See complete classification which follows

2. Surfaces of Double Curvature not surfaces of revolution such as { Ellipsoids with a variable generatrix, Elliptical paraboloids, Elliptical hyperboloids, Serpentines }

Surfaces of Revolution may be either

1. Ruled surfaces such as

1. Single curved surfaces as { Rt. Cir. Cylinder, Rt. Cir. Cone }

2. Warped surface as { The hyperboloid of revolution of 1 sheet. Only case. }

or

2. Double curved surfaces as { Sphere, Ellipsoid { Prolate, Oblate }, Paraboloid, Hyperboloid of 2 sheets }

CHAPTER XII

PLANE SURFACES

80. A plane surface is represented by the plan and elevation of any one of its sections and of its edges. While the drawing may be made by making this section in any plane it is usually taken for convenience in H or V; in whatever plane the section lies it is called a *base*.

When the base lies in H or V it will be apparent that it is formed by the intersections of the traces of the planes of the faces, the edges of the base in reality being portions of the traces. The edges of the surface are the intersections of the planes of the faces. Thus, in Fig. 74, the planes R, S, and T form the faces of the surface, a triangular prism, and the intersections of these planes, XA, YC, ZB, form the edges of the surface; the triangle XYZ, made by the intersections of the V traces of these planes, forms the V base; the triangle ABC made by the intersections of the H traces forms the H base.

81. To assume a point on a plane surface any line as MN, Fig. 74, may be assumed in the plane R, S, or T by Article 25, . then any point as O on this line will lie on the surface of the prism.

82. In case the given base of the surface does not lie in H or V the problem may be solved either by extending the surface so as to obtain a base in either H or V, or in a plane parallel to H or V, or by using the plane of the given base as an auxiliary plane of projection. As a rule the problem may be solved more readily by finding a base in H or V.

NOTE. A *section* is the intersection obtained by cutting a surface by a plane. If the plane of the section is perpendicular to the axis of the surface the section is called a *right section;* other sections are, in general, *oblique sections*.

83. Proposition 18.   Given the size and location of the right section of any prism to find the plan and elevation of the surface.
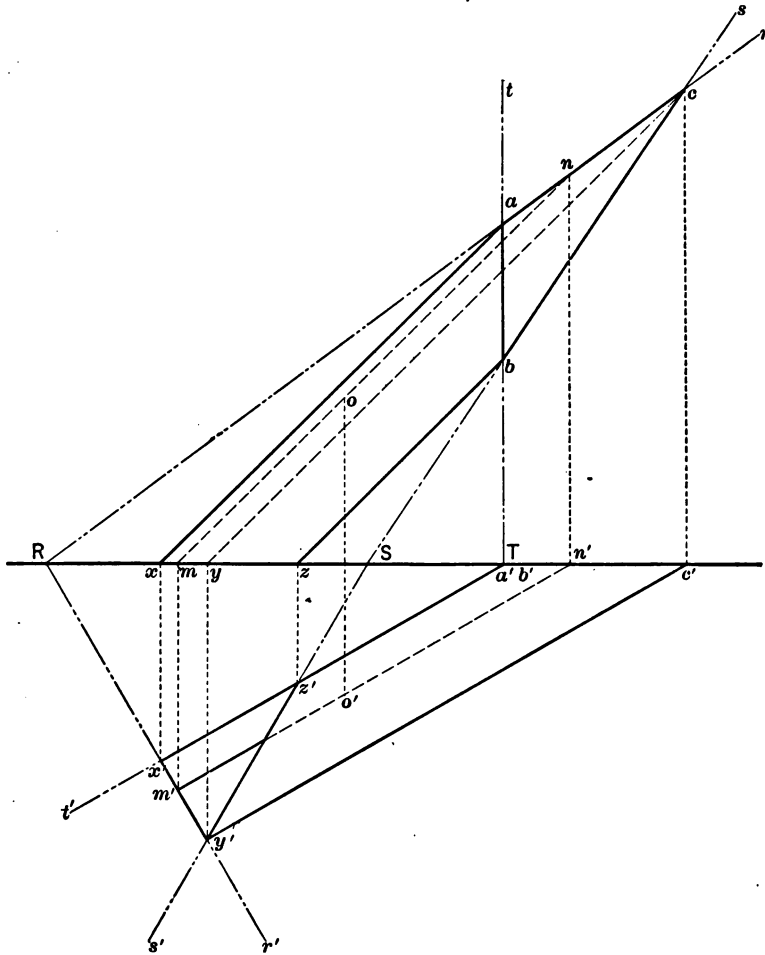
FIG. 74.

Discussion.   Since the right section lies in a plane perpendicular to the axis the edges of the surface will be perpendicular to this plane.   If, then, through the corners of the given right section lines be drawn perpendicular to the plane of this section these lines will be the edges of the surface and the points where

these edges pierce H and V will determine the corners of the bases in H and V.
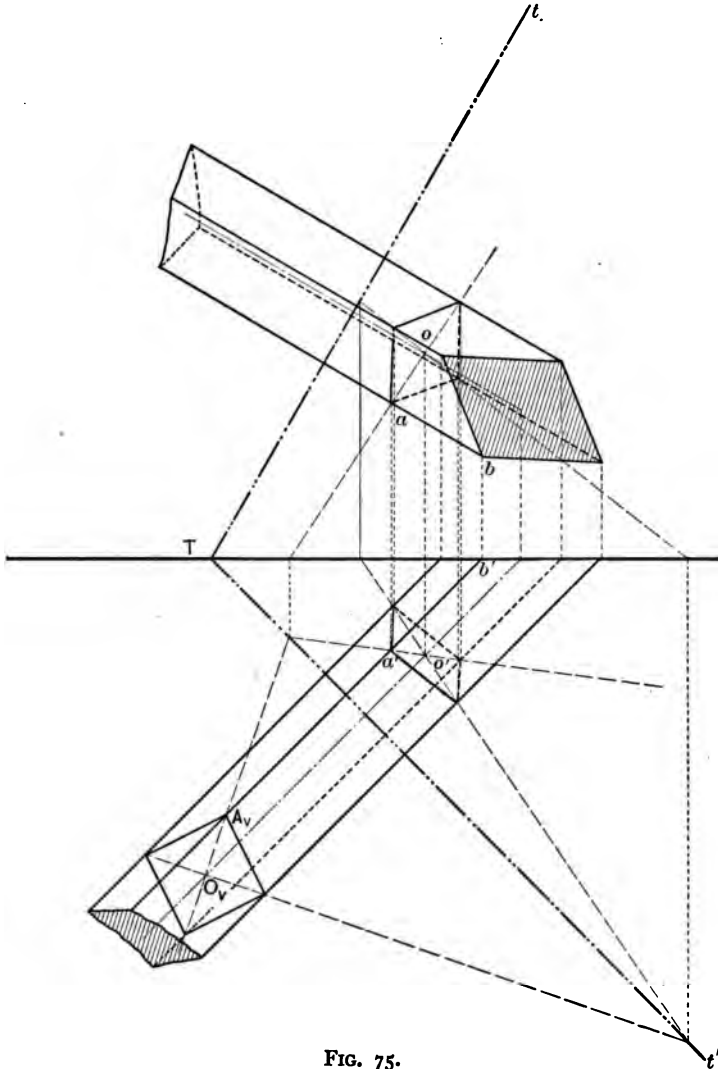
FIG. 75.

Construction. In Fig. 75, T is the plane of the given right section whose center is at the given point O. The section is to be a square of given dimensions. To find the plan and elevation

of this section O is revolved into V at O_v; around this position of O the square may be constructed in its true size and revolved back to its true position in plane T. (Article 37.) Through the corners of the square in its true position draw the lines which form the edges of the surface. Their plans and elevations will be perpendicular to the corresponding traces of the plane T, as ab and a'b'. Extend these lines until they pierce H, — in this case, — and join the piercing points. The area thus enclosed will be the base of the surface in H. In the figure this area has been cross hatched, thus representing the object to be a solid.

84. Proposition 19. Given the plan and elevation of a plane surface to find the true shape and size of its right section.

Discussion. Pass a plane perpendicular to the axis of the surface; this will be the plane of the right section. Find where the edges of the surface pierce this plane; these points when joined will determine the right section. Revolve this area into H or V about the H or V trace of the section plane; this will give the true shape and size of the required section.

Construction. The figure is to be drawn according to the above discussion.

85. Proposition 20. To find the section cut from any plane surface by any plane.

Discussion: First Method. Find the intersection of the plane of each face of the surface with the given cutting plane. Since each of the lines thus found is common to both the surface and the cutting plane the area enclosed by these intersections will be the section cut from the surface by the plane.

Construction. In Fig. 76 the surface is assumed to be a rectangular pyramid cut by the plane sSs'. The plane R is the left face of the solid and cuts the plane S in the line AB; also the plane T, which is the plane of the right face of the solid, intersects the plane S in the line CD. In like manner the lines AD and BC may be found. Since these four lines are common to both the surface and the plane S the area they enclose will be the section cut from the pyramid by the cutting plane S. If the true size of this area is desired it may be found by revolving ABCD into H or V about the corresponding trace Ss or Ss'.

Discussion: Second Method. Find where each edge of the surface pierces the cutting plane. Since these points all lie in one plane and are common to the surface and the cutting plane they will, when joined, form the required intersection.
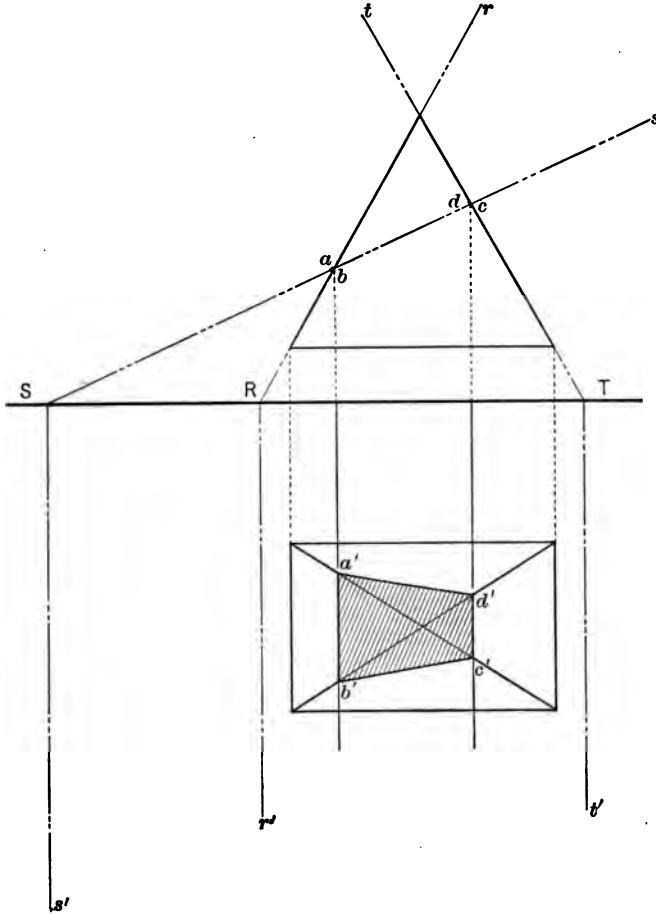
FIG. 76.

Construction. In Fig. 77, O is the apex of a solid hexagonal pyramid whose base is in a plane parallel to H. By Article 56 find where each edge pierces the cutting plane S, as at X, Y, etc. Join these points in order and the area thus enclosed will be the section cut from the surface by the plane S. If the true size of

the section is required it may be found by revolving it into H or V. In this figure the section is revolved into H about the trace sS (Article 36), and its true size is shown by the hatched area at $X_H Y_H$, etc.
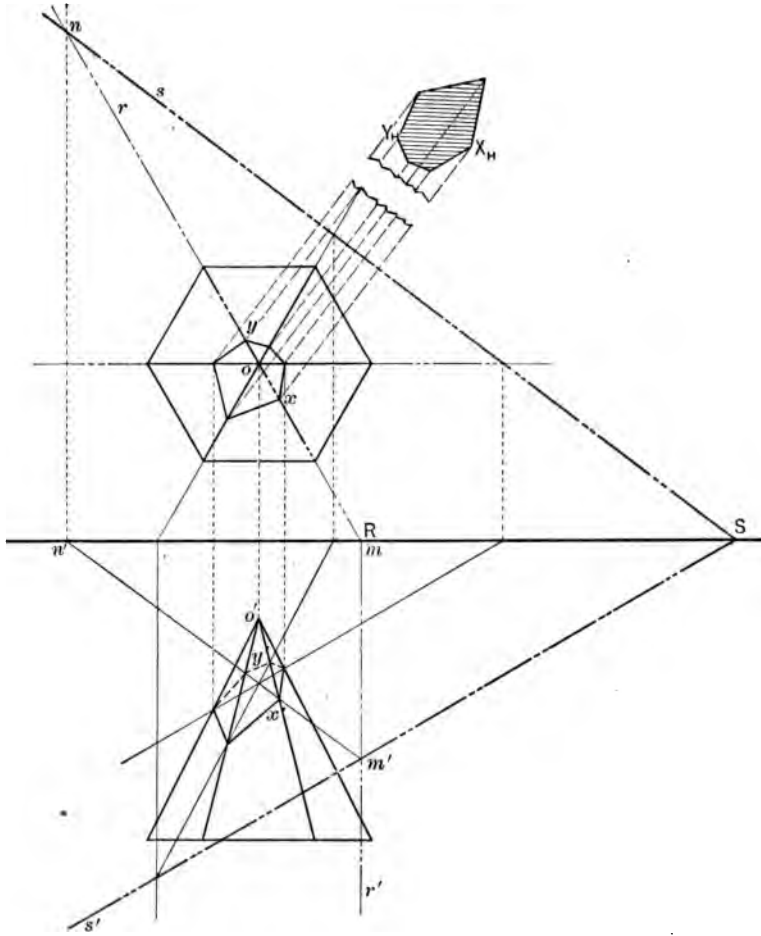
FIG. 77.

86. Proposition 21. To develop any plane surface.

Discussion. Find the true size of each face of the surface and lay these faces out on a plane with their common edges coinciding.

Construction. In Fig. 78 is shown a hood such as might be used over a forge. To make a development of it first find the true size of the edge OA. At any convenient place on the paper lay off OA equal to a'o'', then with A as a center and AD as a radius strike an arc and with O as a center and OD equal to OA as a radius strike a second arc intersecting the first at D. The area AOD is the true size of the back of the hood. In similar fashion using OD as a common edge construct the area OCD
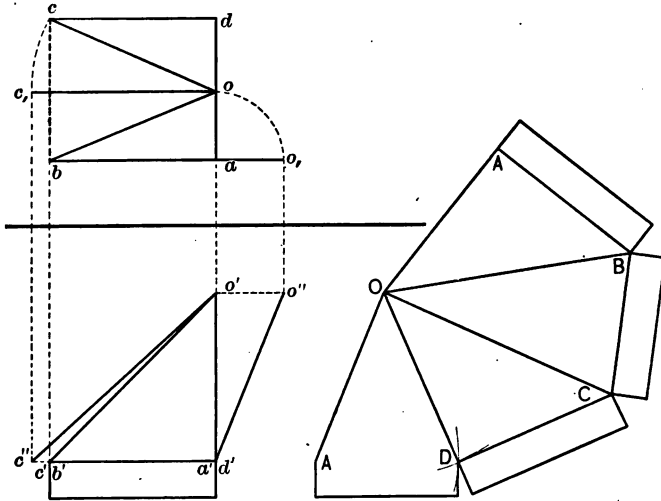
FIG. 78.

equal to the side of the hood, and continuing this process each face of the hood may be laid out. The lips of the hood, or the portions lying below the line ABCD are laid off by constructing rectangles with one side equal to AD, DC, CB, and AB and the other side equal to the depth of the lip. These rectangles are of course joined to the hood along the lines AD, DC, etc.

In case the edges of the surface to be developed do not meet in a point, as in Fig. 78, or are not parallel, as in the case of a prism, the development may be made as shown in Fig. 79. The method used is the same in principle as that used in Fig. 78, modified slightly to suit the altered conditions.

Construction. Lay off the area BCHG making BC = bc, CH = c'h', BG = b'g' and GH = gh. This will be the back of

the piece. Now with C as a center and a radius equal to c'e' strike an arc, and with H as a center and h'e' as a radius strike a second arc intersecting the first at E. This point E is the position of E in the development of the face CDEH. Now to find the position of D strike arcs with C as a center and c'd' as
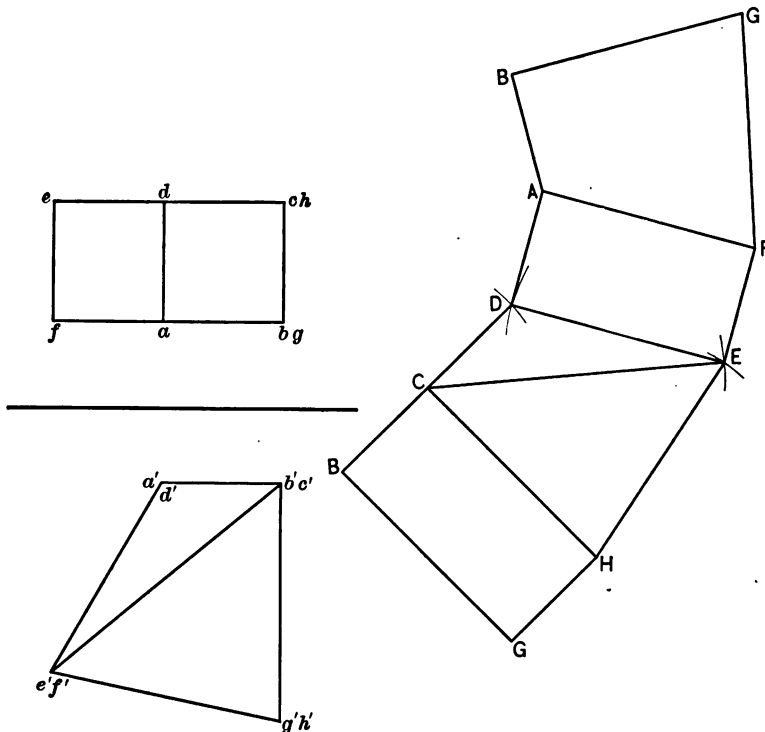
FIG. 79.

a radius, and with E as a center and e'd' as a radius. These arcs intersect at D in the figure and thus the side CDEH may be developed by dividing it into triangles and locating the corners as shown. The layout for the remaining portion of the pieces will be apparent from an inspection of the figure.

PROBLEMS ON PLANE SURFACES

Special Proposition. Given the axis, the apex, and the size and shape of the right section at a given point on the axis, to find the plan and elevation of a pyramid. Write the discussion and draw a figure demonstrating the above proposition.

167. The center line of a timber 4″ square pierces H 8″ in front of V and inclines 45 degrees to H and 30 degrees to V. Draw a plan, elevation, and end view of the timber.

168. The center line of a timber 4″ square pierces H at O (o″; −6″; o″) and pierces V at Q (18″; o″; −12″). Draw a plan and elevation of the timber and show its intersection with H and V.

169. The point O is o″; −6″; o″. The point Q is −8″; o″; −8″. The line OQ is the center line of a rectangular pipe 1″ by 3″ in section. Draw a plan and elevation of the pipe and find the true size of the hole it makes in passing through the profile plane located at −5″; 90; 90.

170. The point O is −2″; −4″; o″. The point Q is 2″; o″; −4″. The line OQ is the center line of an hexagonal prism whose base in H is a regular hexagon with sides 1″ long. Draw a plan, elevation, and end view of the prism and show its base in V.

171. Make a drawing showing how to trim the ends of a 2″ by 4″ stick so that it will fit against V making an angle of 45 degrees, will fit against H making an angle of 30 degrees, and will have a center line 18″ long between these points.

172. The plane T is o″; −60; +60. The point O in this plane is −3″; x″; −3″ and is the center of a regular hexagon with 1″ sides which lie in plane T. Draw a plan and elevation of the prism perpendicular to V which could cut this hexagon from the plane.

173. The point O is o″; −3″; −3″. The point Q is 6″; −1″; o″. The line OQ is the center line of an hexagonal prism whose base in P is a regular hexagon with 1″ sides and center at O. Draw a plan, elevation, and end view of this prism and find the true shape and size of its right section.

174. The base of a pyramid is a hexagon with 1½″ sides in H and its center at the point o″; −3″; o″. The apex of the pyramid is at Q (−4″; −1″; −3″). Draw a plan and elevation of this pyramid and find its right section at a point 2″ from the apex.

175. The plane T is o″; −60; +45. The point O is in this plane at −4″; −2″; x″. This point O is the center of a rectangle 2″ by 3″ whose sides incline at 45 degrees to the H trace of plane T. The rectangle is the base of a quadrilateral pyramid whose altitude is 3″. Show three views of the pyramid.

176. A trough is built up of a 2″ by 8″ piece nailed to two 2″ by 6″ pieces. Draw the plan, elevation, and end view of a section 18″ long which slopes 30 degrees to H and V.

177. Make a development of a hopper whose opening is a rectangle 18″ by 36″ and whose 12″ by 12″ outlet is formed by cutting the hopper by a plane inclining 30 degrees to the horizontal. The distance between the centers of the openings is 18″.

178. Make the pattern for a chute 12″ square in section to connect the opening in a bin wall 48″ above the floor with an opening in the floor 36″ from the bin wall.

179. An air duct of 6″ by 8″ pipe follows the line of intersection of the walls and ceiling of a room with its 8″ side against the ceiling. Lay out patterns for two-piece elbow needed in the corners.

180. Lay out the pattern for a pipe to connect a 12″ square hole in a floor with a bin wall. The center line of the pipe inclines 30 degrees to the floor, 45 degrees to the bin wall, and is 72″ long.

181. Design the transition piece to connect a 12″ square air duct with a 4″ square air duct. The two ducts lie on the floor of a building against the wall with their ends 6″ apart.

182. Design a ventilating hood for the corner of a room. The opening of the hood is to be 18″ by 24″, the outlet is to be in the corner into a 9″ by 12″ pipe, and the hood is to be 16″ deep with 4″ lips.

183. A 12″ square ventilating pipe runs along the intersection of the ceiling and front wall of a room. Design the elbow which will take the pipe down the side wall from the corner at an angle of 30 degrees with the ceiling.

184. Make the patterns for an elbow turning an angle of 105 degrees using 4″ by 6″ pipe with the plane of the angle in the direction of the long dimensions of the pipe.

185. Make the patterns for an elbow turning an angle of 60 degrees using 4″ by 6″ pipe with the plane of the angle in the direction of the short dimension of the pipe.

186. A water duct runs down the side of a building at an angle of 15 degrees with the horizontal and turns around the corner of the building. Lay out patterns for an elbow to make the turn using 3″ square pipe.

CHAPTER XIII

CYLINDRICAL SURFACES

87. A cylindrical surface is represented by the plan and elevation of some curve of the surface and of the limiting elements of the surface. For convenience this curve is usually taken in H or V and is therefore the locus of the points in which the elements of the surface pierce H or V.

88. In a cylinder this curve is called a base and for convenience it is taken as a rule either in H or V, or in a plane parallel to H or V. Since the base of a cylinder must be a closed curve, cylinders will be either circular or elliptical in section, or the section will be some irregular closed plane curve such as, for example, would be formed by cutting a corrugated tin pipe. When a cylinder with a *circular* right section is inclined to H or V its bases in those planes will be ellipses; when the cylinder is *elliptical* in right section and is inclined to H or V its bases will, in general, be ellipses, although if the angle of the inclination be properly adjusted they may become circles. Thus, in Fig. 80 is a cylinder with a circular base in H. This cylinder must be elliptical in section. In Fig. 81 is a cylinder with an elliptical base in V. This cylinder may be either circular or elliptical in section, according to the angle its axis makes with V.

89. Cylinders are defined by their bases. Thus: a cylinder with a circular base; a cylinder with an elliptical base, etc. A *right cylinder* is one whose elements are perpendicular to the plane of its base. Such cylinders may be either circular or elliptical in section and are generally named accordingly, as a right circular cylinder, or a right elliptical cylinder.

90. To assume a point on the surface of a cylinder it is necessary only to assume an element of the surface and on this element locate the point. In Fig. 80, OP is the axis of a cylinder whose base is a circle in H. Assume the element XY parallel, of course,

to the axis and piercing H in the base at Y.   Any point on this
element will be a point on the surface of the cylinder.   The
element projected at x'y' has two plan views, one at xy and the
other at qr.   Either of these is correct as one is on one side of
the cylinder and the other is directly opposite on the other side.
This same thing is shown in Fig. 81.   If the point Q were given
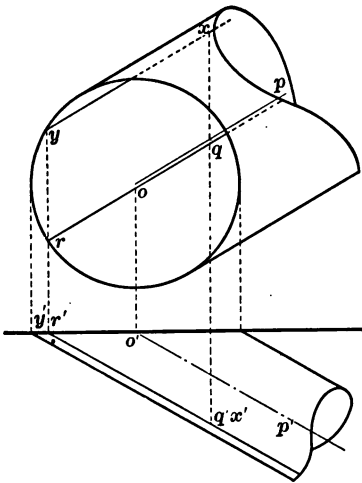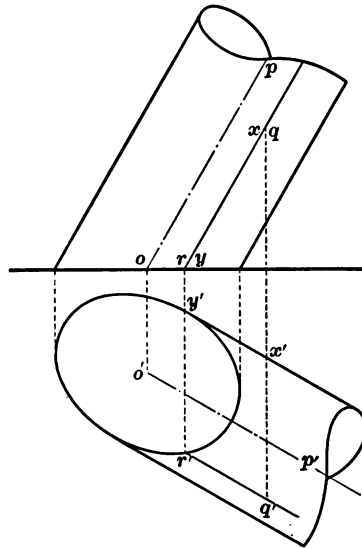
FIG. 80.   FIG. 81.

by its plan view only, two V projections would be possible, one
at q' and one at x', since the element through Q has two possible
V projections.   In given problems it will always be stated on
which element to take the point.

91. Proposition 22.   Given the axis and right section of any
cylinder to find its plan and elevation.

Discussion: First Method.   Revolve the axis parallel to H
or V.   In this position draw the right section.   Through the
ends of the axes of the curve of the right section draw elements;
these limiting elements will pierce H or V at the ends of the axes
of the base.

Construction.   In Fig. 82, OQ is the given axis and the given
right section is a circle whose diameter is dc long.   Revolve OQ

parallel to V; in this position it is projected at oq, and $o'q''$. At any convenient point P place the given right section. Its elevation will be a straight line, $a''b''$, since the axis is parallel to V; and its plan view will be an ellipse whose major axis will be
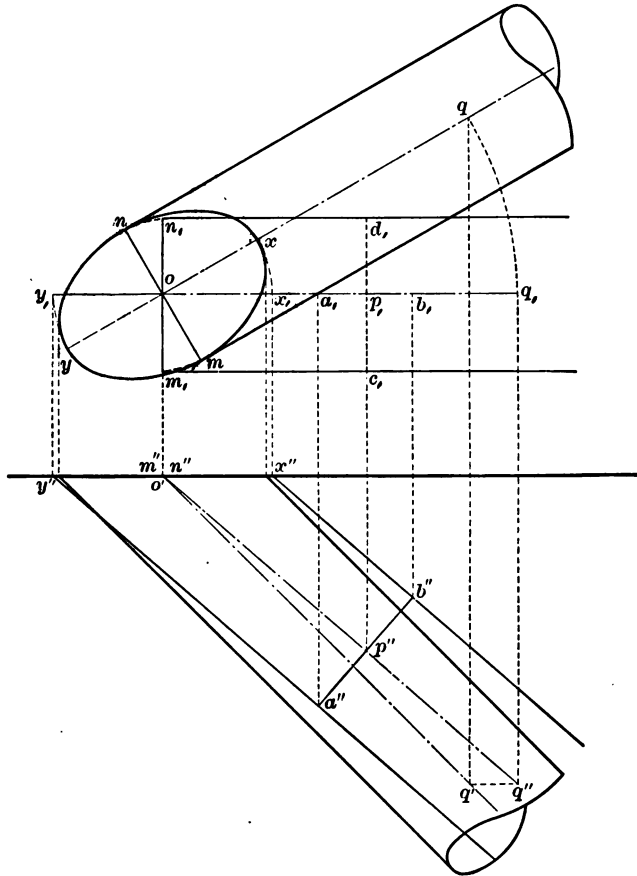
FIG. 82.

$d_{,}c_{,} = a''b''$ and whose minor axis will be $a_{,}b_{,,}$ or the projection of $a''b''$ on H. Through A and B draw lines parallel to OQ in its revolved position; these will be limiting elements and will pierce H at $y_{,}$ and $x_{,,}$ the ends of the major axis of the base in H. Likewise draw the limiting elements through D and C; these

pierce H at n, and m, respectively and locate the ends of the minor axis of the base in H.   Now revolve the axis back to its given position OQ and locate the true position of the axes of the base.   Since the angle that OQ makes with H remains constant the axes of the base remain constant and in the same relation to OQ.   Therefore, x, will revolve to x, y, to y, n, to n, and m, to m.   Construct the ellipse of the base and draw the projections of the limiting elements.

Discussion: Second Method.   Pass a plane perpendicular to the given axis; this plane contains the given right section.   Construct the projections of the given right section, draw the limiting elements, and find where they pierce H or V.   These points will give the axes of the base in H or V.

Construction.   Through any convenient point P, Fig. 83, pass a plane T perpendicular to the given axis OQ.   (Article 48.) Revolve the point P into H about Tt.   With this point P as a center draw the given right section in its true size.   When this circle is revolved to its proper position in plane T it will be projected as an ellipse and the diameters AC and BD will become the major and minor axes of this ellipse as it is projected in H. Find the projections of A, B, C, and D by Article 37 and through these points draw the projections of elements of the surface. These elements pierce H at yx and mn, which points are respectively the ends of the major and minor axes of the base. Draw the ellipse determined by these points and find the projections of the limiting elements.

92. Proposition 23.   To find the curve cut from any given cylinder by any given plane.

Discussion.   Pass planes through the cylinder parallel to its axis.   These auxiliary planes cut elements from the surface of the cylinder and lines from the cutting plane.   These lines intersect the elements in points common to both the cylinder and the cutting plane.   If a sufficient number of auxiliary planes be used enough points will be obtained to plot the curve of intersection.

Construction.   When the cylinder is perpendicular to H or V. In Fig. 84 the given circular cylinder with its base in H is cut by

a plane T.   Through the axis perpendicular to plane T pass
auxiliary plane R.   This plane cuts two elements from the cylin-
der at D and C, and a line DC from the plane T.   This line in-
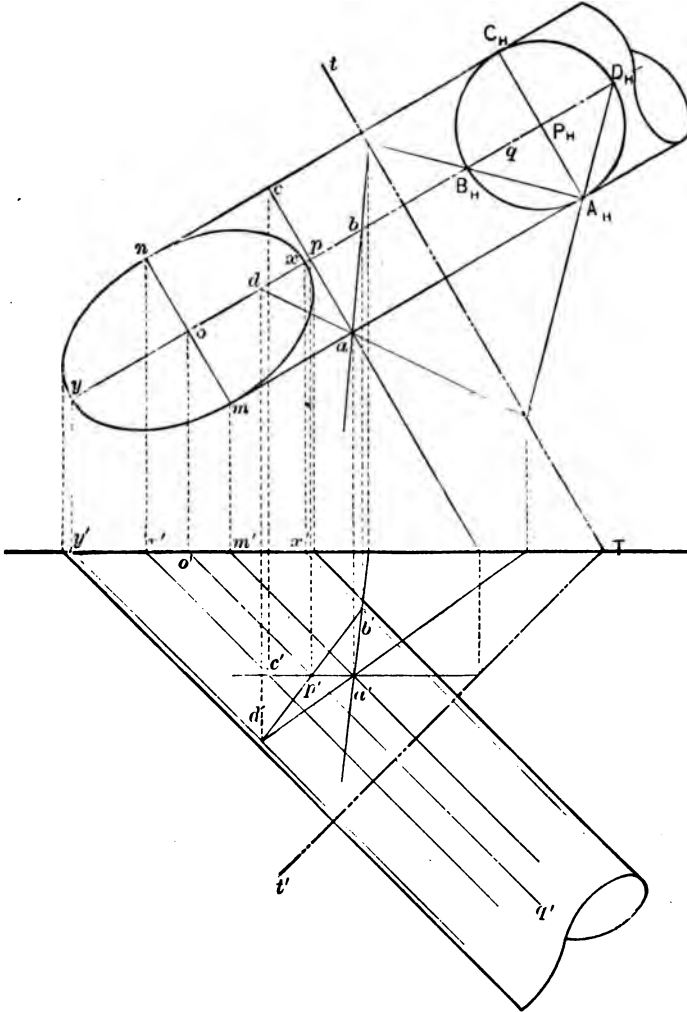
FIG. 83.

tersects the two elements at D and C, two points of the required
curve.   In like manner the plane U cuts two elements at A and B
and a line AB from the plane T.   This line intersects the two ele-

ments at A and B.   From an inspection of the drawing it will be
seen that C is the point where the plane T enters the cylinder and
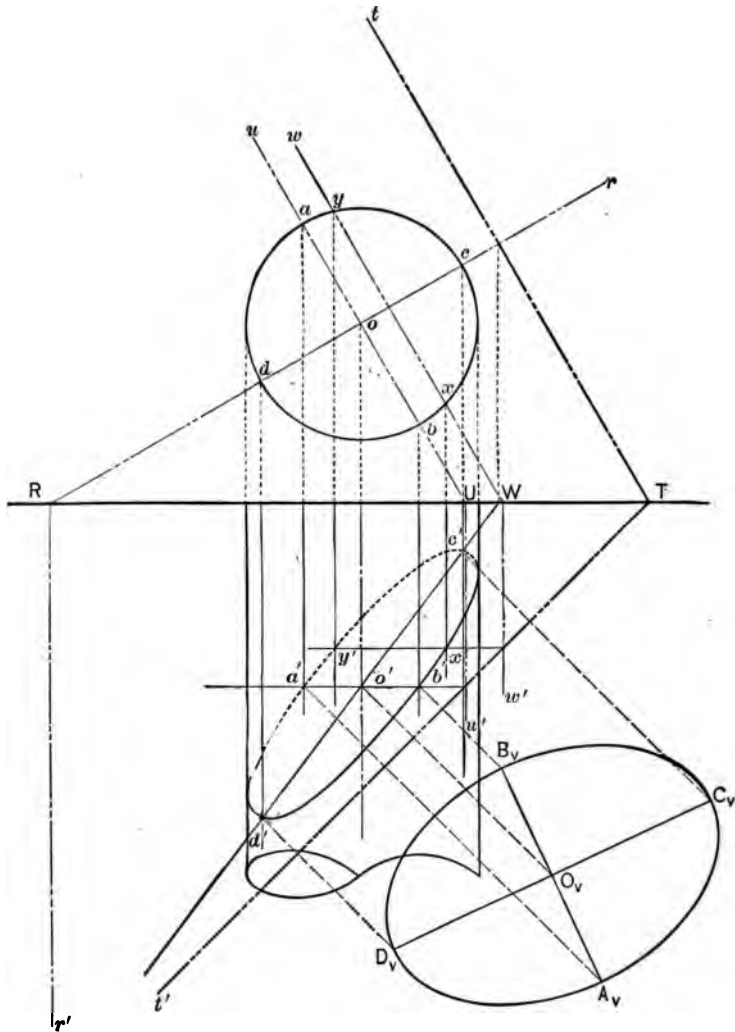
FIG. 84.

the point D is where the plane leaves the cylinder; therefore C and
D are the two points on the curve of intersection farthest apart,
or the ends of the major axis of the curve.   Since the auxiliary

plane U contains the line AB and is perpendicular to the plane R the two points A and B will be the ends of the minor axis of the curve of intersection. Now if AB and CD be revolved into V about Tt′, the true size of the curve of intersection may be found as shown. If the projections of this curve of intersection be required they may be found by revolving the ellipse $A_VB_VC_VD_V$ to its original position, or by passing more auxiliary planes parallel to U, as W, and locating points as in plane U.

Construction. When the cylinder is oblique to H or V. In Fig. 85 is shown a cylinder oblique to H with circular base in H. This cylinder is cut by the plane T as in the case just discussed. Pass planes, such as R, U, W, through the cylinder parallel to its axis. Each plane, except the plane tangent along the limiting elements, cuts two elements from the cylinder and a straight line from the cutting plane T. These lines are of course parallel as shown at AM, BN, CQ, and intersect the elements of the cylinder in points of the curve of intersection, as A, B, C, and D. If a number of auxiliary planes are passed a sufficient number of points will be determined to plot the curve of intersection. In finding the true size of this curve the points may be revolved into V or H about the corresponding trace. In this case the curve is revolved into V about the V trace taking the position $A_VB_VC_VD_V$.

93. It must be remembered in working problems of this character that the more points there are found on the curve the more accurate will be the final result. In the figures given as illustrations only a few points have been located in order to avoid confusion in the drawings.

It must also be remembered that only by solving the problem in an orderly fashion can accurate results be secured. It is not wise to attempt short cuts, and in the end it will be found most satisfactory to pass the auxiliary planes *one* at a time and to determine *all* of the points in each plane before passing to the next. Any attempt to pass all of the auxiliary planes, then to draw all of the elements these planes cut from the cylinder, and then to locate all of the lines cut from the cutting planes results either in throwing the problem into a hopeless confusion of lines,
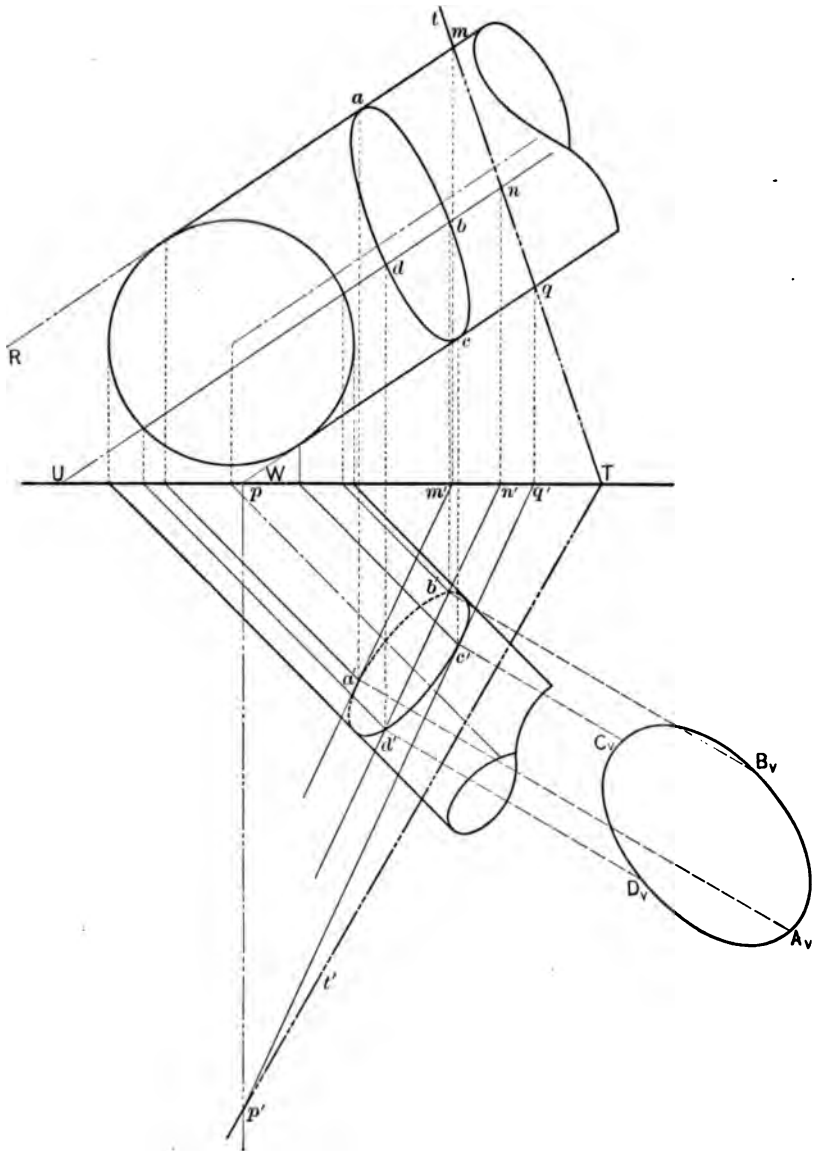
FIG. 85.
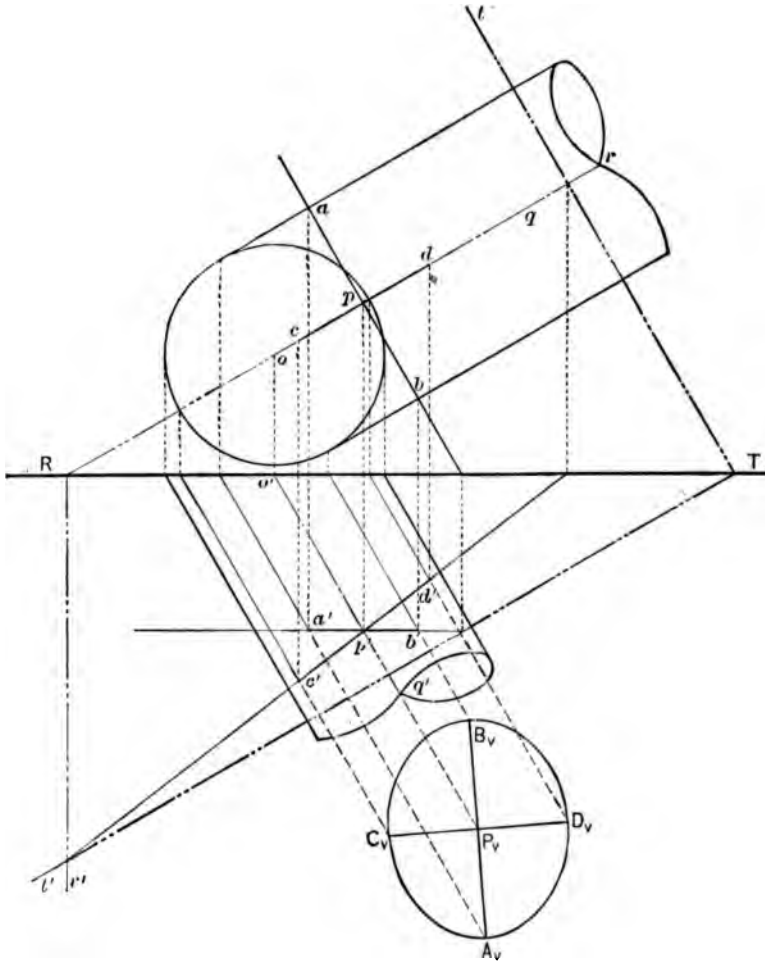
FIG. 86.

or else requires more painstaking work to avoid this tangle than the method suggested. This condition is true of all problems of this character.

94. Proposition 24. Given any cylinder to find its right section.

Discussion. Pass a plane through the cylinder perpendicular to its axis and find the curve of intersection; this will be a right section of the cylinder.

Construction. If both the true size and the projections of the right section are required the problem may be best solved by Proposition 23. If, however, only the true size of the right section is required the solution in Fig. 86 will be found more simple. Let OQ be the axis of the given cylinder and T the perpendicular plane containing the right section. The axis OQ pierces the plane T (Article 56) at P; this will be the center of the right section. Now draw a line AB through P parallel to the H trace of T; this line will be the longest line which can be drawn between two elements of the surface; hence it is the major axis of the curve of intersection. To find the minor axis draw CD through P perpendicular to the trace tT. Revolve AB and CD into V about the V trace to A_VB_V and C_VD_V; with these lines as axes construct the ellipse which will be the right section of the cylinder.
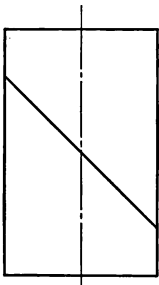
FIG. 87.

95. It will have been noted in connection with Article 92 that the intersection of a right cylinder with an oblique plane is a curve which is symmetrical about its center lines. This fact is utilized in constructing cylindrical elbows so that the pipe section may remain uniform while the elbow turns an angle.

In Fig. 87 is shown the plan and elevation of a cylinder cut by an oblique plane. Since this curve of intersection is symmetrical the upper part of the cylinder may be turned through 180 degrees to the position shown in Fig. 88 and the two parts of the cylinder will still join on the same curve of intersection while the axis turns an angle of 90 degrees. Fig. 89 shows how this same

principle may be applied to elbows with more than one section. If the problem be to draw a four section elbow turning an angle of θ degrees, Fig. 90, draw two lines making the given angle with each other, as AO and DO, and then divide the angle by drawing BO and CO so that the angle

$$BOA = \frac{\theta}{3}.$$
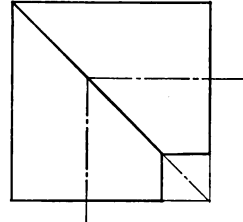
FIG. 88.

Find the points 1, 2, 3, and 4 all equidistant from O and with these points as centers lay off on AO, BO, CO, and DO the diameters of the right sections of each part of the elbow. The limiting elements may be drawn through the ends of the diameter of the right section.
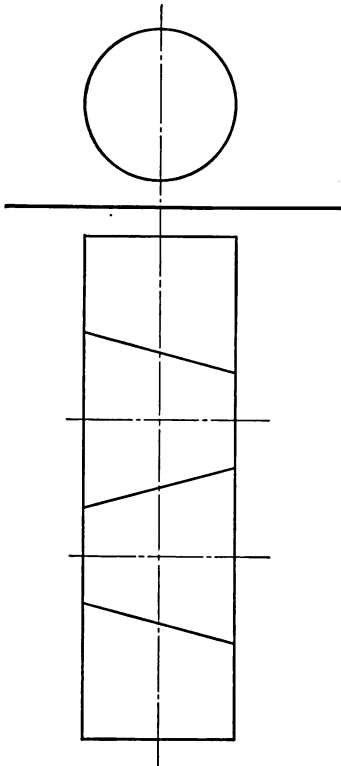
96. Proposition 25. To develop any cylinder.

Discussion. Since the elements of a cylinder are perpendicular to the plane of the right section the curve of the right section will roll out in the development into a straight line to which all the elements of the surface are perpendicular. Therefore, lay off a line equal in length to the right section and erect perpendiculars to this line. If the distances between these perpendiculars be made to equal the distances between the elements along the curve of the right section and the perpendiculars be made equal in length to the elements the result will be a development of the cylinder.

FIG. 89.

Construction. In Fig. 91, the cylinder taken is part of an elbow with a 90 degree turn. With the bow dividers set off on

the base the distances 1–2, 2–3, 3–4, etc., equal and small enough so that the chord of the arc is equal, practically, to the arc itself, and through these points draw elements of the cylinder. The last step, 11–12, may not be equal to the others but this does not matter. Now draw a line 1–1 (it will be convenient to draw this line with the T square a continuation of the projection of the base of the cylinder) and divide it as shown so that the divisions are equal to the distances laid off on the base of the cylinder.
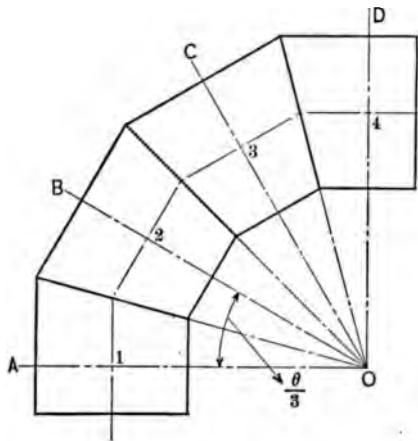
FIG. 90.

Through these points erect perpendiculars equal in length to the elements. Through the points a, b, c, d, etc., thus found draw a curve. The area bounded by the points a–1–1–a
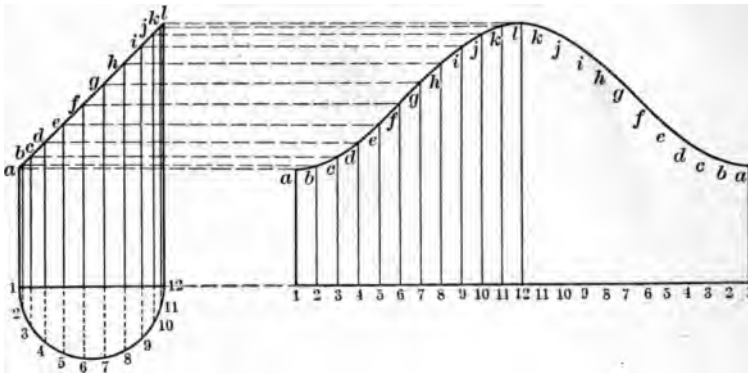
FIG. 91.

is the developed cylinder. It will be obvious from the figure that since the development is symmetrical on each side of the center line 12–1 only one-half of the base need be divided in order to develop the whole cylinder.

PROBLEMS ON CYLINDERS

Special Proposition. Given a right circular cylinder parallel to the ground line and an oblique plane to find the curve cut from the cylinder by the plane. Write the discussion and draw a figure demonstrating the above proposition.

187. The axis of a cylinder pierces H at a point $1\frac{1}{2}''$ in front of V. A second point on this line, $4''$ from the first point, is $3''$ in front of V and $2\frac{1}{2}''$ above H. A right section of this cylinder is a circle $2''$ in diameter. Draw a plan and elevation of this cylinder and show the true shape and size of its base in H.

188. The point O is $0''$; $-2''$; $0''$. The point P is $4''$; $-4''$; $-3''$. The line OP is the axis of a cylinder whose right section is a circle $2''$ in diameter. Draw a plan and elevation of this cylinder and find its base in H.

189. The axis of a cylinder pierces H at a point $2''$ back of V and pierces V at a point $3''$ below H. Between these two piercing points the axis is $6''$ long. Draw a plan and elevation of the cylinder showing its base in H as a circle $1\frac{1}{2}''$ in diameter and find its base in V.

190. The axis of a cylinder pierces V $4''$ below H, and inclines 30 degrees to V and 45 degrees to H. Draw a plan and elevation of the cylinder and find its base in H when its base in V is a circle $2''$ in diameter.

191. A $3''$ circular air pipe crosses the corner of a room. Its axis cuts the side wall $5''$ from the corner and the front wall $6''$ from the corner; both of these points are $8''$ below the ceiling. Show the true size of the holes which will have to be cut for the pipe.

192. The axis of a right circular cylinder $3''$ in diameter pierces H $2''$ back of V and is perpendicular to H. Find the traces of a plane which will show the angle at which this cylinder has to be cut in order to get an ellipse whose axes are $4''$ and $3''$.

193. The point O is $0''$; $-0''$; $-1\frac{1}{4}''$. The point P is $0''$; $-4''$; $-1\frac{3}{4}''$. This line is the axis of a right circular cylinder whose diameter is $2\frac{1}{2}''$. The cylinder is cut by a plane located at $4''$; 135; 240. Show a plan and elevation of the curve of intersection and its true size.

194. The point O is $0''$; $-2''$; $-0''$; the point P is $0''$; $-2''$; $-4''$. The plane T is $-4''$; $+30$; -45. Find the true shape and size of the curve cut from the cylinder whose axis is OP and whose diameter is $2''$.

195. Draw the plan and elevation of a cylinder whose base is a $2''$ circle in H with its center $2''$ below the G. L., and whose axis is inclined so that its plan makes 45 degrees with the G. L. and its elevation makes 60 degrees with the G. L. Cut this cylinder by a plane which makes an angle of 75 degrees with the axis. Find the plan and elevation of the curve of intersection and its true size.

196. The point O is $0''$; $-3''$; $0''$; the point Q is $3''$; $-5''$; $-4''$. The line OQ is the axis of a cylinder whose base in H is a circle $2\frac{1}{2}''$ in diameter.

The plane T is 4"; −45; +45. Draw a plan and elevation of the intersection of the plane and cylinder and find its true size.

197. The plane S is o; −45; −60. The point O lies in this plane at −1"; −2"; x", and is the center of a 2" circle whose plane is S. Draw the plan and elevation of the cylinder of which this circle is a right section and find one of its bases in H or V.

198. The point O is −2"; −3"; o". The point Q is 2"; o"; −5". The line joining OQ is the axis of a circular cylinder whose diameter is 2". Draw the plan and elevation of the cylinder showing its bases in H and V.

199. The axis of a cylinder whose right section is a 2" circle pierces H 3" back of V and pierces V 4" below H. Between these two points the axis is 7" long. Draw a plan and elevation of the cylinder and find its base in P.

200. A right circular cylinder whose diameter is 2½" is cut at an angle of 60 degrees to its axis. Make a pattern for this cylinder assuming the longest element of the cylinder to be 4".

201. A right elliptical cylinder is cut off by a plane inclining to the axis at such an angle that the section cut is a circle. The right section of the cylinder is an ellipse 2" by 3" and its longest element is 5". Lay out a pattern for this cylinder.

202. Make a development of the air pipe in Problem 191 and lay out on the development the line of intersection between the pipe and the walls.

203. Make the patterns for a four piece elbow of 6" pipe. The elbow turns an angle of 90 degrees, with a radius of center line 12" long.

204. The axis of a 3" circular cylinder pierces H 4" back of V and pierces V 3" below H; between these two piercing points the axis is 5" long. Make a pattern of the cylinder between its H and V bases.

205. A line of 8" pipe slopes at the rate of 4' in every 10' and runs due northeast. At a point o"; −6"; o" the pipe meets the horizontal floor it is to drain. Make a pattern showing how the last section of pipe must be cut to fit into the floor.

206. Make the patterns for a five piece elbow of 8" pipe. The pipe turns an angle of 105 degrees, with a radius of 16".

CHAPTER XIV

CONICAL SURFACES

97. A conical surface is represented by the projections of some curve of its surface and the limiting elements which meet at the apex. As in the case of cylinders this curve is usually taken in H or V and is the locus of the piercing points of the elements.

98. A cone is represented by the projections of any one of its sections and the limiting elements. For convenience the section taken is usually the base in H or V, and the projections of the limiting elements are found by joining the limiting points in the base with the projections of the apex. Thus, in Fig. 92, the base is an ellipse in H whose center is at O, and the apex of the cone is given at Q. To find the projections of the limiting elements for the plan views of the cone tangents are drawn from q to the ellipse; and to find the projections of the limiting elements for the elevation of the cone lines are drawn from q' to the extreme points of the elevation of the base. It will be obvious that the limiting elements of the elevation are not projections of the limiting elements of the plan view.

99. Cones are identified by their bases. Thus: a cone with a circular base or a cone with an elliptical base, etc. As in the case of cylinders a cone with a circular base will be elliptical in section when its axis is inclined to the plane of the base, and a cone with an elliptical base may be either elliptical or circular in section according to the angle of inclination.

A *right cone* is one whose axis is perpendicular to the plane of the base. Right cones may be either circular or elliptical in section and are named accordingly, as: a right circular cone, or a right elliptical cone.

100. To assume a point on the surface of the cone either projection of a point may be assumed on the corresponding pro-

jection of some element and the other projection of the point found by locating it on the other projection of the same element. Thus, in Fig. 92, the projection y′ is assumed on the projection of the element q′r′ and the projection y then will be found on qr. It will be noted, as in the case of cylinders, that there are two elements of the cone whose elevations coincide with this line, one being QR and the other QP, so that it is possible to have two points on the cone whose elevations coincide at x′, one being Y
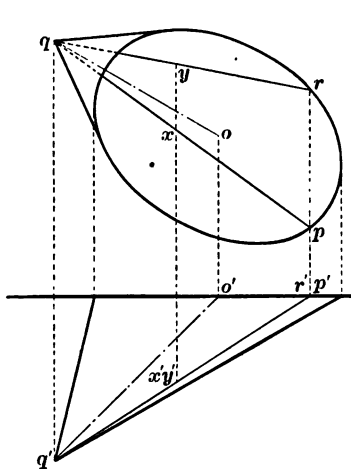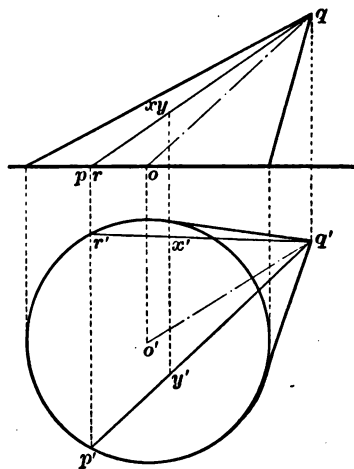
FIG. 92.   FIG. 93.

and the other X. In given problems the data are usually given which determine which point is intended, otherwise two solutions to such problems are possible.

In Fig. 93 a similar problem is shown. In this case the plan view of the point is assumed and its elevation located at one of the points x′ or y′, depending on which side of the cone the point is to be taken.

101. Proposition 26. Given the axis of a cone, the size and location of its right section to find the plan and elevation of the cone.

Discussion. If the cone be revolved parallel to a plane of projection, the given right section may be drawn in its given location. If, then, elements be drawn through this section from

the apex the base of the cone in H or V may be found by finding where the elements pierce H or V.

Construction. In Fig. 94, OQ is the given axis, and P the given point on this axis where it pierces the given right section. The right section in this construction is taken as a circle whose diameter is equal to x″y″. Revolve OQ parallel to V; in this
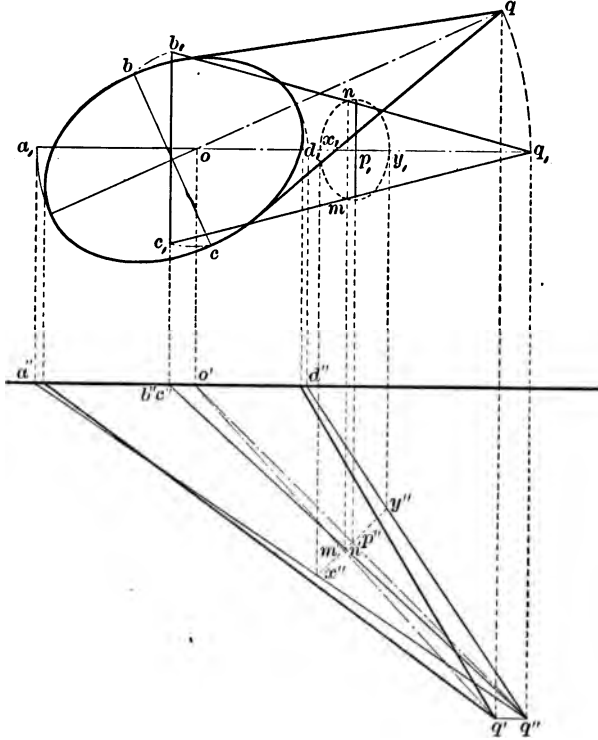
FIG. 94.

position it is projected at oq, and o′q″ and P falls at p, and p″. Draw x″y″ equal to the given diameter of the right section per-pendicular to o′q″; this line will be the elevation of the right section, since OQ is parallel to V, and the ellipse x,y, will be the plan view. Now through x″ and y″ draw the elements projected on V at q″d″ and q″a″, and on H at q,d, and q,a,. These elements pierce H at a, and d,, which points will be the ends of the major axis of the base in H. To find the minor axis of the

base bisect a,d, by the line whose direction is b,c,. The **length** of the line is as yet unknown but whatever this may be, since it is perpendicular to V, the line will be projected on V at the point b″c″. Join b″c″ with q″; this will be the elevation of the two elements which pierce H at the ends of the minor axis. Now these elements cut the curve of right section in its revolved position at M and N, and if through these points elements be drawn they will pierce H at c, and b,, or the ends of the minor axis. The axis may now be revolved to its original position, and the true position of these axes located. The curve in the base may be constructed and the limiting elements drawn.

102. Proposition 27. To find the curve cut from any given cone by any given plane.

Discussion. Through the apex of the cone pass auxiliary planes intersecting the cone and the given plane. These planes cut elements from the cone and lines from the given plane. Since these elements and lines lie in common planes they will intersect in points which will be common to both the cone and the given plane. If a sufficient number of auxiliary planes be passed enough points may be found to plot the curve of intersection.

Construction. When the axis of the cone is perpendicular to H or V and its base is a circle. In Fig. 95, sSs′ is the given plane cutting the right circular cone whose apex is at Q. Through the apex Q pass the plane R perpendicular to Ss. This plane cuts from the plane S the line AB, which line pierces the surface of the cone at A and B. Since this line is the longest line which can be drawn common to both the cone and the plane S it will be the major axis of the curve of intersection. Now through P, the middle point of AB, draw CO in plane S and perpendicular to AB. This line CO pierces the surface of the cone at the points C and D. To find C and D pass a plane uUu′ through the apex Q and the line CO. This plane cuts two elements from the cone which intersect the line CO at the points C and D where CO cuts the surface of the cone. Since it is the shortest line which can be drawn common to both cone and plane S, and since it is perpendicular to AB, CD will be the minor axis of the

curve of intersection. To find its true size revolve the axes of the curve into V about s'S and construct the ellipse $A_VB_VC_VD_V$.

If the projections of this curve of intersection are required they may be found by passing other auxiliary planes, such as R, through the apex of the cone and finding points by the method used to find points A and B.
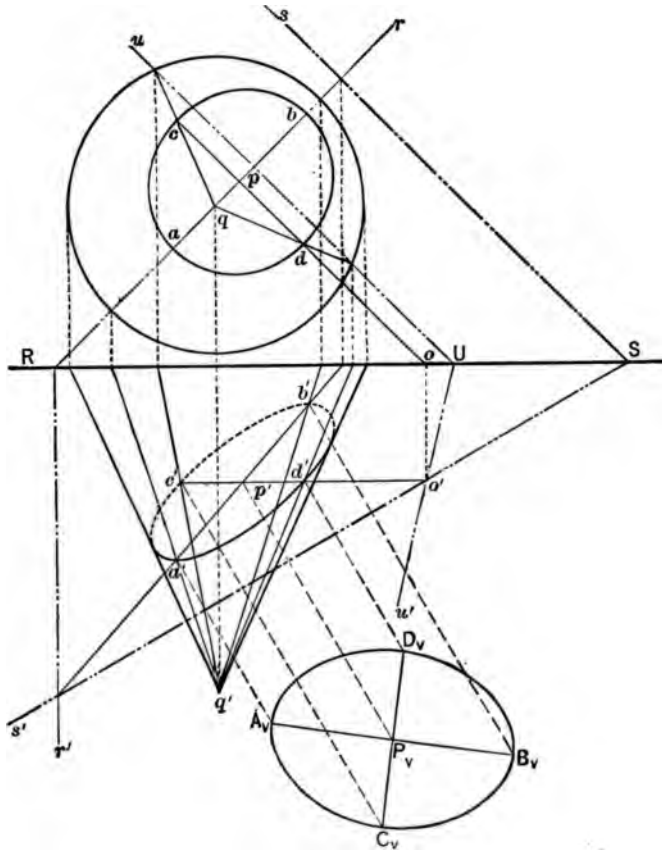
FIG. 95.

Construction. When the axis of the cone is oblique to H and V, the base being any plane curve.

In Fig. 96, S is the given plane cutting the cone whose apex is Q. Pass auxiliary planes R, U, V, etc., through Q perpendicu-

lar to H.   The auxiliary plane R cuts from the cone one element
and from the plane S the line OM.   This line OM intersects this
element at F, one point on the curve of intersection.   In like
manner the plane U determines the point E of the curve.   Now
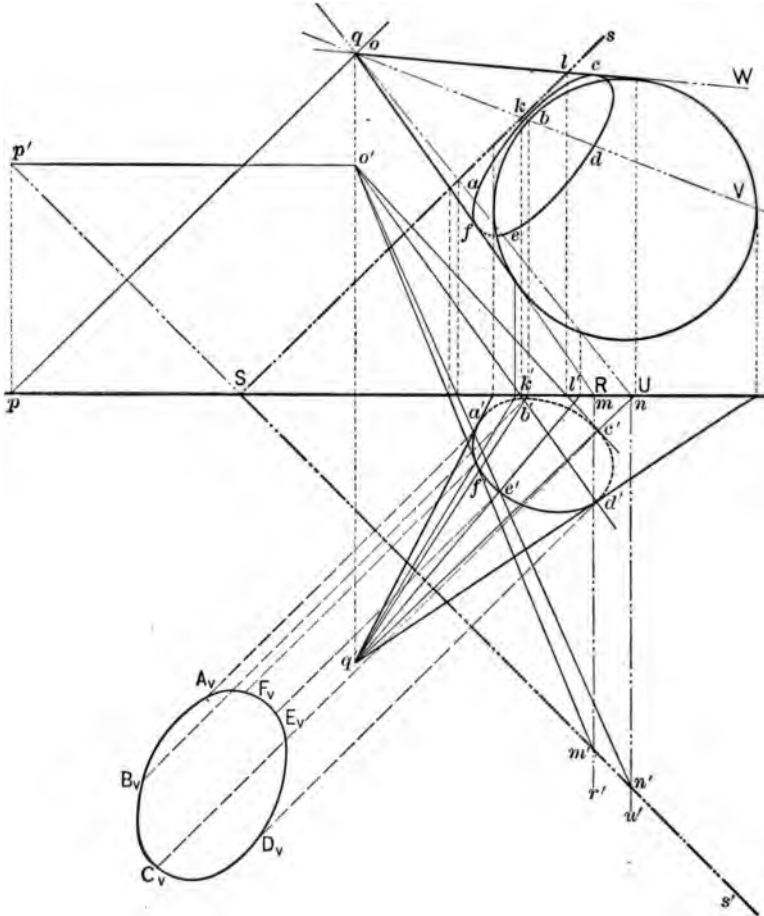
FIG. 96.

in problems of this character it generally happens that the
auxiliary planes run off the paper, as, for example, V, making it
impossible to find points common to the V traces, as m' and n'.
To avoid the difficulty which arises the following method of
finding the intersections of S with the cutting plane is convenient.

Since the auxiliary planes all pass through the apex of the cone and are perpendicular to H, they will have a line in common which passes through Q perpendicular to H. Now this line pierces the given plane S in a point which is common to S and all the auxiliary planes. This point, therefore, is common to all the lines cut from S by the auxiliary planes. Through this point and the intersections of the H traces lines of intersection between the planes may be drawn without using the V traces. This point in the given case is O. To find it locate a point O in the plane S, having its plan view at q (Article 26). Since the line common to all the auxiliary planes is perpendicular to H through the apex its plan view will coincide with q.

Now the auxiliary plane V cuts from the plane S the line OK and from the cone two elements; this line and the elements intersect at B and D on the required curve. In like manner other points may be determined and the curve of intersection drawn. To find the true shape and size of the curve it is necessary to revolve it into H or V about the corresponding trace and to locate a number of points, as $A_V B_V C_V$, etc.

103. Proposition 28. Given any cone to find the shape and size of its right section at any given point.

Discussion. If the cone be revolved parallel to H or V the right section may be drawn through the given point. In this position the axes of the right section will be projected on H or V in their true size.

Construction. In Fig. 97, the given cone has a circular base in V and an apex at O. About the center of the base revolve the cone parallel to H, the projections of the apex falling at $o_{\prime}$ and o''. Now find OQ, the axis of the cone, by bisecting the angle at the apex and on this line locate the given point P, at which the section is to be taken. Through P, pass a plane R perpendicular to the revolved position of OQ. This plane contains the revolved position of the right section, and the points C and D, where the limiting elements pierce R, are the ends of the minor axis. Since CD is parallel to H and the axes are perpendicular, the major axis will be projected on H as a point at $p_{\prime}$ and on V as a line through p'' perpendicular to $c''d''$.

Now through p, draw the projection of the elements which pass through the ends of the major axis; both of these elements are projected in H at $o,q,$ and pierce V at x'' and y''. o''x'' and o''y'' are the elevations of the elements which pass through the ends of the major axis at a'' and b''. a''b'' and c''d'' are ele-
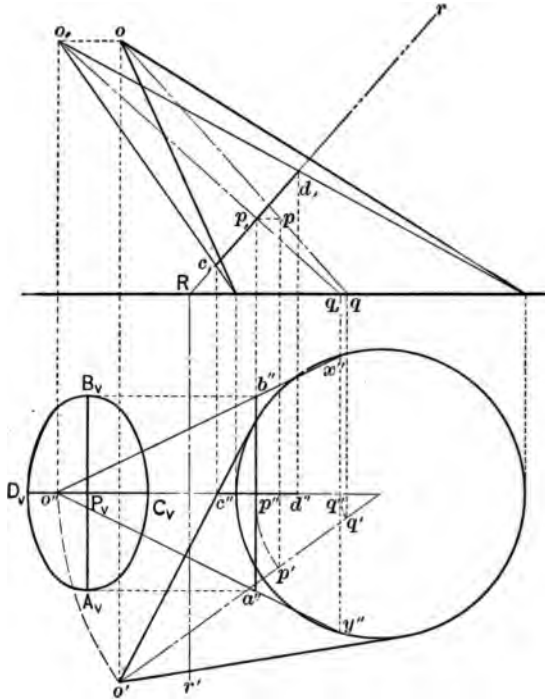
FIG. 97.

vations, then, of the axes of the right section at P, and by revolving them into V about r'R the true size of the curve may be drawn at $A_V B_V C_V D_V$.

104. If the base of the cone is an ellipse whose major axis coincides in direction with the projection of the axis of the cone, this same method may be used to find the right section. In such a case, however, it is possible for the section to be a circle and if the cone inclines so that the plane R cuts a line c,d, from the cone equal to the line a''b'' such will be the case.

When this method will not apply the right section may be

found as in Article 102 by taking the cutting plane perpendicular to the axis of the cone.

105. As in the case of cylinders the fact that an oblique plane cuts a symmetrical curve from a right cone may be utilized in
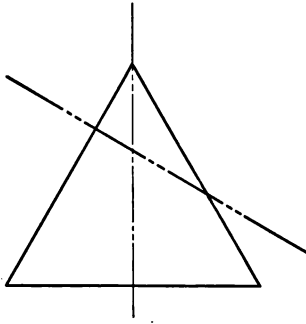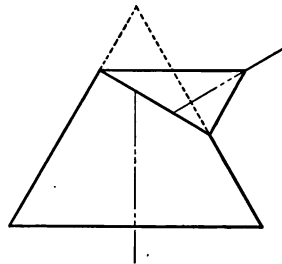
FIG. 98.   FIG. 99.

constructing conical elbows. Fig. 98 shows a typical case when the top of a given cone is cut off, turned through 180 degrees,
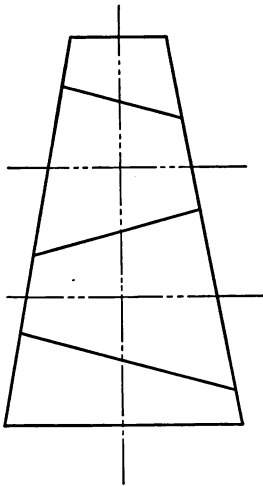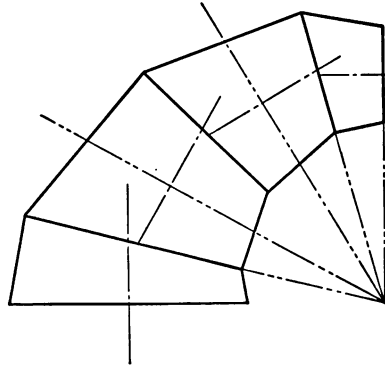
FIG. 100.   FIG. 101.

and set back. The axes of the two sections not only turn an angle but they no longer meet. The reason for the latter point will be clear from an inspection of Fig. 99.

Fig. 101 shows an elbow made up of sections of the same cone, the cone being divided as shown in Fig. 100. Since the axis of the cone does not pierce the plane of each joint in the center of the plane of the joint the axes will have to be offset as shown.

A simple method of drawing such elbows is shown in Fig. 103, which is a design for a ship's ventilator of three conical and one cylindrical sections, whose axes are given by the broken line
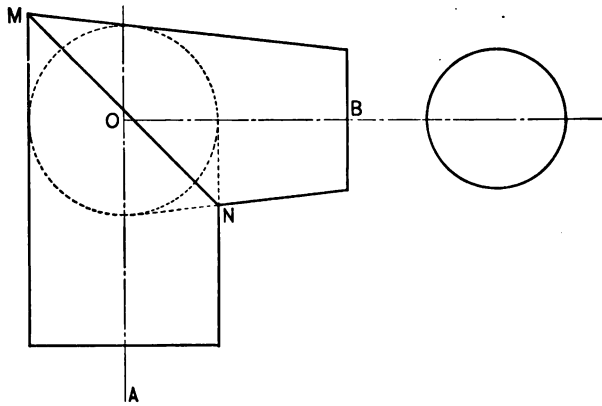
FIG. 102.

ABCDE. The problem is to find the size of the right cones which will fit together with plane joints.

The principle by which the problem may be worked may be stated as:

106. **Proposition 29.** To find the size of a cone or cylinder which will fit another cone or cylinder with a plane joint, the axes and the size of one cone or cylinder being given.

Discussion. Since an oblique plane cuts an ellipse from both a right cone and a right cylinder, the problem is to find where to place this plane so that the sections cut from the surface will be equal. In Fig. 102 the right cylinder whose axis is AO is to form an elbow with a right cone whose axis is BO, and whose right section at B is the circle shown. Now if at O, the intersection of the axes, a sphere be drawn to which the given cylinder is tangent, and a cone be drawn with BO as an axis tangent to the sphere, this cone will join the cylinder in an ellipse whose plane

is shown as the joint in the elbow, MN. Since the cone and the cylinder are each tangent to a common sphere they will have one curve of intersection in common.

Construction. To apply this principle to the drawing of the ventilator cowl made up of conical sections, spheres must be drawn at B, C, and D as shown in Fig. 103. The diameter of the sphere at B must be equal to the diameter of the cylinder; the diameter of the spheres at C and D are to be assumed so that
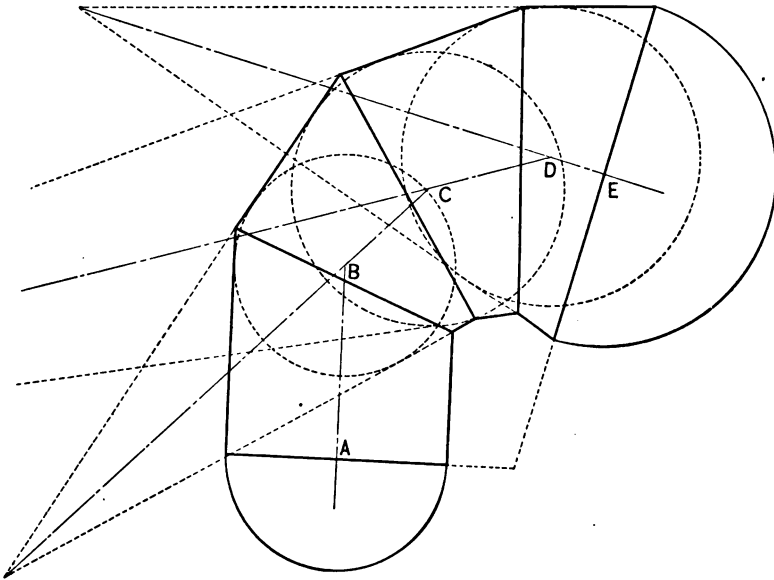
FIG. 103.

the increase in size in the sections of the cowl is fairly uniform. Tangent to these spheres are drawn the limiting elements of the cones and where these pairs of limiting elements intersect will be the joints. The opening at E is usually given so that the limiting elements of the last section are drawn from the ends of the given diameter of the opening at E to the sphere whose center is at D.

107. Proposition 30. To develop any given cone.

Discussion. Draw lines from a common point — the apex — equal in length to the elements of the cone, and spaced according to their distances apart on some curve of the surface.

Construction. When the given cone is a right circular cone. In Fig. 104 the base of the given cone is a circle in H, the apex is at O, and the cone has been cut off along the plane of a'g'. Since the cone is right all the elements from the apex to the base are
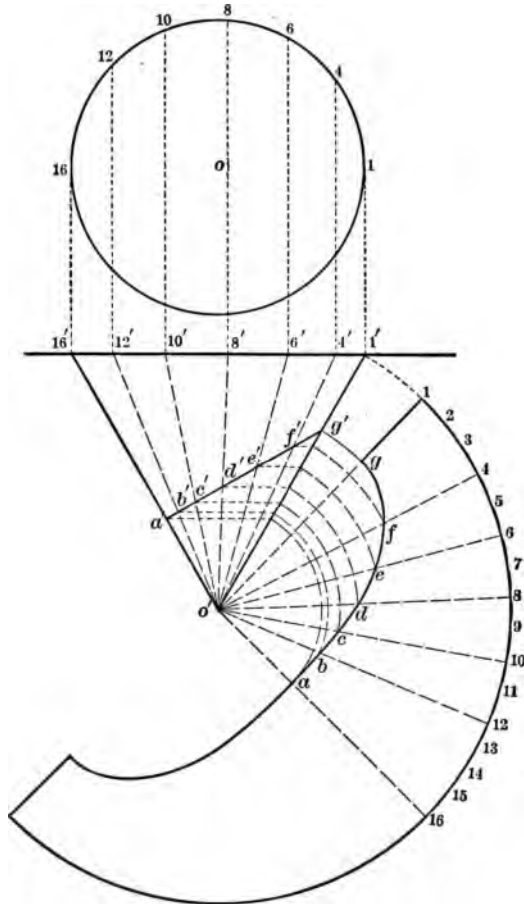
FIG. 104.

equal in length; therefore, with o' as a center and o'1' as a radius describe the arc 1——16, etc., and at any convenient point draw the line o'g1 equal to o'g'1'. This line will be the first element in the development. Now divide the circle of the base into a convenient number of parts and step these off on

the developed base as 1, 2, 3, 4, 5, 6, etc. Find where each of the elements pierces the plane through a'g' and find the true distance from O to each point. Lay this distance off on the development
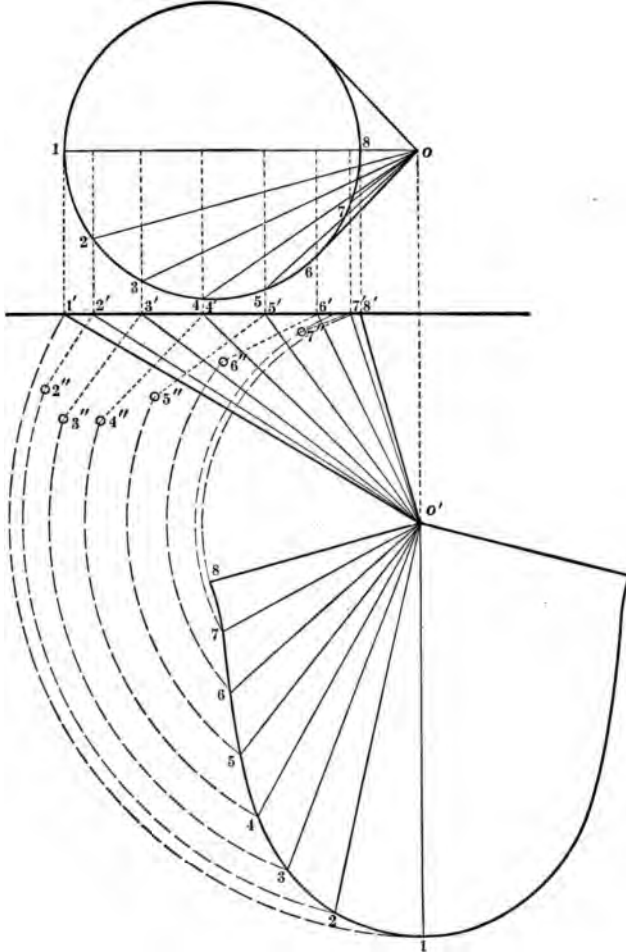
FIG. 105.

as at g, f, e, d, etc. In the figure only a few points have been worked out to avoid the confusion of lines but in practice the elements should be spaced about as shown by the points 1, 2, 3, 4, etc.

Since the development will be symmetrical about a center line O16 the two parts may be constructed simultaneously.

Construction. Method of Triangulation. When the given cone is oblique. In Fig. 105 the given cone is taken with its axis parallel to V with its base in H and its apex at O. Lay off o'8 equal to element O8. Now with the true length of the arc 7-8 as a radius and 8 as a center strike an arc, and with o' as a center and the true length of O7 as a radius strike a second arc intersecting the first at 7. o'7 then is the position in the development of the element O7. In practice the distance 8-7 must be made small enough so that the triangle o'87 will lie on the surface of the cone; in the figure these lines are taken far apart to avoid confusion.

In like fashion with o'7 as one side construct triangle o'76 equal to the triangle O76 on the cone, and proceeding this way the entire surface of the cone may be developed by dividing it up into small triangles and laying these out in their true size and relation.

This method is the method by which most developments are made. It will apply to the development of any object whose surface is capable of being divided into triangles.

PROBLEMS ON CONES

Special Proposition. Given a cone and a line to find where the line cuts the surface of the cone. Write the discussion and draw a figure demonstrating the above proposition.

207. The axis of a cone is $3\frac{1}{2}''$ long. It pierces H $1\frac{1}{2}''$ in front of V and inclines 45 degrees to H. The apex of the cone is $3''$ in front of V. A right section of the cone $2''$ from the apex is a circle $1''$ in diameter. Draw a plan and elevation of the cone and find its base in H.

208. The point O is $0''$; $-3''$; $0''$. The point Q is $-3''$; $-5''$; $-4''$. The line OQ is the axis of a cone whose right section $4''$ from the apex Q is a circle $1\frac{1}{2}''$ in diameter. Draw the plan and elevation of the cone and show its base in H.

209. The axis of a cone pierces H $3''$ in front of V. The apex of the cone is $3''$ above H and $2''$ behind V and the axis from the apex to the H piercing point is $7''$ long. The base of the cone in H is a circle $2\frac{1}{2}''$ in diameter. Find the base of the cone in V.

210. The point O is $-3''$; $-3''$; $0''$. The point Q is $2''$; $-3''$; $-4''$. The H base of this cone whose axis is OQ is a circle $2\frac{1}{2}''$ in diameter. Find the base of the cone in P.

211. The point O is $-2''$; $0''$; $-3''$. The point Q is $2''$; $-4''$; $-3''$. The base of the cone whose axis is OQ is a $1''$ circle in $P = 0''$; $90°$; $90°$. Find the base of this same cone in V when Q is the apex.

212. A conical lamp shade whose height is $8''$ is hung from the center of a $9'$ by $12'$ room with its apex $7'$ $6''$ above the floor. The base of the shade is $12''$. Find the lighted areas on the walls and floor of the room made by the cone of light reflected from the shade.

213. The axis of a right cone is perpendicular to H $2\frac{1}{2}''$ behind V; the base of this cone in H is a $3''$ circle and its altitude is $4''$. Cut this cone by three planes: one at an angle to its axis cutting all of the elements; one parallel to the axis; and one parallel to one element of the cone. Show the true shape and size of each of these curves of intersection and investigate its properties.

214. Draw a plan and elevation of the cone in Problem 213 and cut the cone by any plane oblique to both H and V. Show the true shape and size of the curve of intersection.

215. The point O is $0''$; $-0''$; $-2\frac{1}{2}''$. The point Q is $0''$; $-2\frac{3}{4}''$; $-2\frac{1}{2}''$. The base of the cone whose axis is OQ is a circle $2\frac{1}{2}''$ in diameter in V. The cone is cut by plane $T = 3\frac{1}{2}''$; -30; -60. Find the true shape and size of the curve of intersection and its plan and elevation.

216. The point O is $3\frac{3}{4}''$; $-2\frac{1}{2}''$; $0''$. The point Q is $1\frac{5}{8}''$; $-4\frac{5}{8}''$; $-3\frac{3}{4}''$. The base of this cone is a circle $3''$ in diameter in H. The cone is cut by a plane which is $0''$; $+30$; -60. Find the true shape and size of the curve of intersection.

217. The point O is $3\frac{1}{2}''$; $-2\frac{1}{2}''$; $-0''$. The point Q is $0''$; $-2\frac{1}{2}''$; $-2\frac{1}{2}''$. The base in H of this cone is a circle $2\frac{1}{2}''$ in diameter. The plane T is $0''$; -60; $+45$ and cuts the cone. Find the true shape and size of the curve of intersection.

218. Draw the plan, elevation, and left end view of a right cone whose base is a $1\frac{1}{2}''$ circle lying in plane $T - 5\frac{1}{2}''$; 60; 30; with its center at $O -3\frac{3}{4}''$; $1\frac{3}{4}''$; x'' and whose altitude is $3''$.

219. A cone of revolution is located in the third quadrant with its base in plane $T -2\frac{1}{2}''$; $+60$; $+30$ with its center at the point $O -5''$; x''; $-1\frac{7}{8}''$. The base is a circle $2''$ in diameter and the altitude is $3\frac{1}{2}''$. Draw three views of the cone.

220. Lay out the patterns for a conical hopper. Diameter of large opening $12''$; diameter of small opening $4''$; distance between openings $6''$.

221. A $20''$ smoke stack passes through a roof which inclines 30 degrees with the horizontal. A conical canopy forms the flashing between the roof and stack. Make a pattern of this canopy with the shortest element $6''$ long.

222. Lay out a pattern for the cone in Problem 216 between the base and the intersection with the plane.

223. The point O is o″; 3″; o″. The point Q is o″; −2; +4″. **The** line OQ is the axis of a cone whose base in H is a 3″ circle. Make a pattern for that portion of the cone lying between H and V.

224. Make a pattern for a sink strainer. This is in the form of one quarter of a cone whose opening is a 16″ circle and whose height is 8″.

225. Make the patterns for a conical elbow of three sections to turn an angle of 90 degrees. Diameter of large opening 10″; diameter of small opening 4″; radius of center line 12″.

226. Design a conical elbow of four sections for a 120 degree turn to connect a pipe of 12″ diameter with another pipe of 6″ diameter, radius of center line 20″.

CHAPTER XV

INTERSECTION OF SURFACES

108. Proposition 31. Given two plane surfaces to find their lines of intersection.

Discussion: First Method. Find where the edges of one surface pierce the planes of the faces of the other surface. These points when joined in proper order determine the line of intersection.

Construction. In Fig. 106 the right hexagonal prism is intersected by a rectangular prism whose edges are oblique to H and V. Find where these edges pierce the faces of the hexagonal prism and join the points in the proper order. In the given case the rectangular prism pierces the hexagonal prism, thus giving two intersections: the entering one at ABCD and the departing one at EFGH. The prisms have been assumed as solids in the drawing.

Discussion. Second Method. Find the intersections of the planes of the faces. These lines will be the edges of the intersection of the two surfaces.

Construction. In Fig. 107 a rectangular hopper, base in H 1–2–3–4 and apex O, intersects a rectangular pipe. The face of the hopper, 1–4–O, lies in the plane rRr′; and this plane intersects U — the plane of the upper face of the pipe — in the line PQ. The portion of PQ between the edges O4 and O1 — or AD — is one side of the opening in the pipe made by the hopper. In like manner CB may be obtained by finding the intersection of plane U with plane vVv′; and with these four points, A, B, C, D, the intersection may be drawn.

109. Proposition 32. To find the intersection of two given cylinders.

Discussion. Pass auxiliary planes through the cylinders cutting elements from each. These elements, being in a common

plane, intersect in points common to both cylinders, or points
on the required curve of intersection.
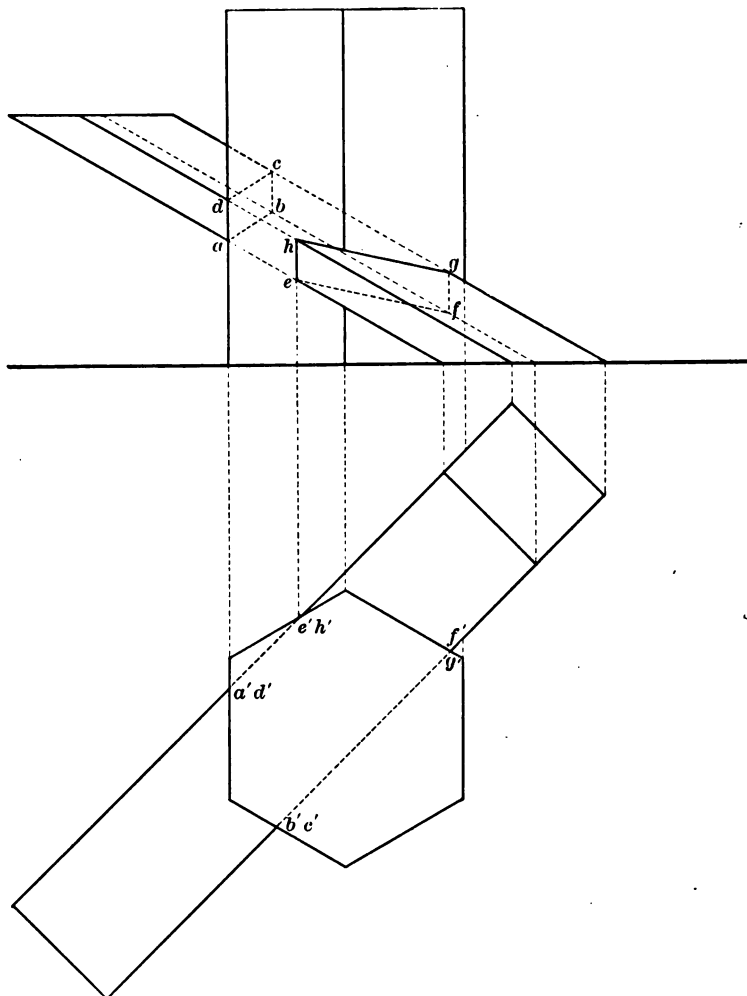
FIG. 106.

Construction: First Case. When the bases of the given
cylinders lie in the same plane.

In Fig. 108 one cylinder is perpendicular to H and the other
oblique to H and V; both have circular bases in H. Pass plane
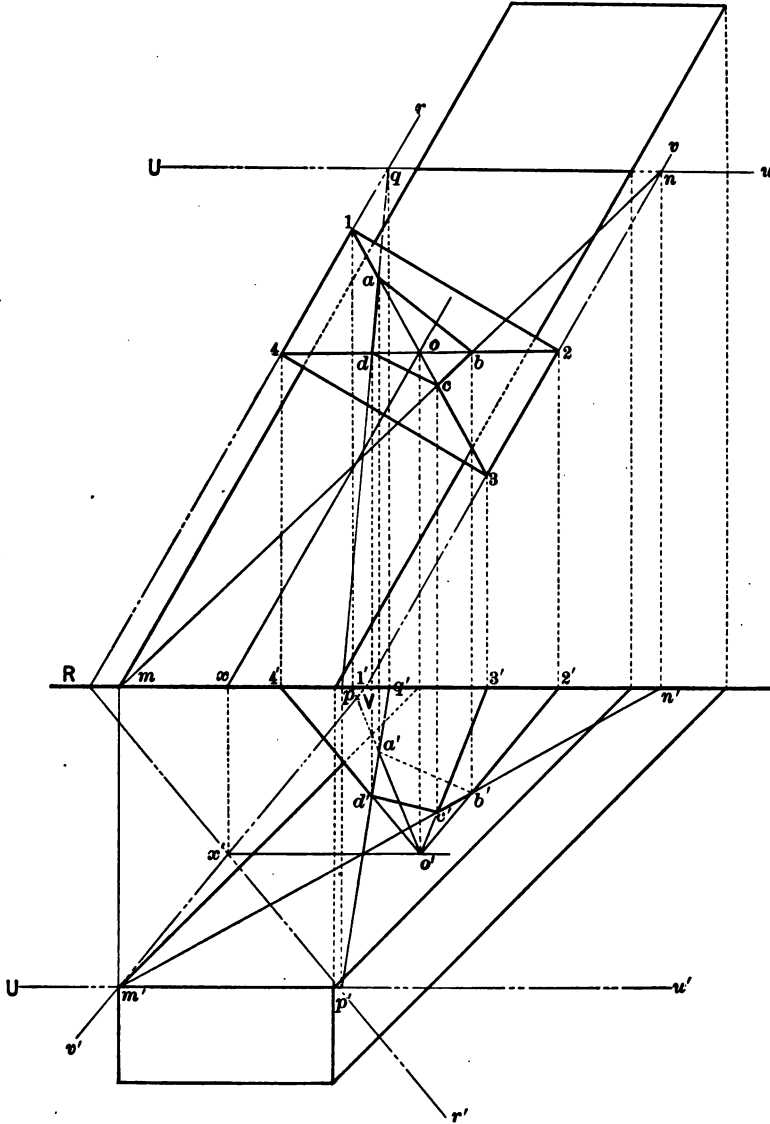
FIG. 107.

R through both cylinders cutting elements from each. **These** elements intersect at A, B, C, and D, as shown. If enough auxiliary planes similar to R are passed a sufficient number of points may be obtained to draw a curve of intersection.
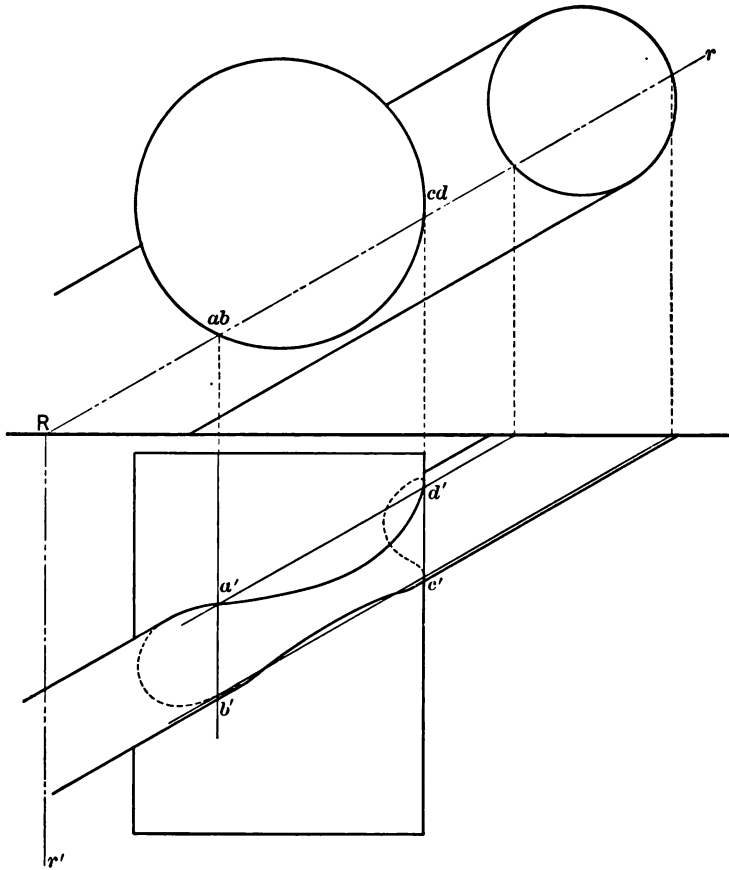
FIG. 108.

Construction: Second Case. When the bases of the given cylinders are in different planes.

In Fig. 109 the larger cylinder has a circular base in H and is parallel to V; the smaller has a circular base in V and is parallel to H. Pass auxiliary planes, as S, through the cylinders parallel to the axis of each and cutting elements from each. These ele-

ments intersect in points of the curve of intersection, as D and B. To find the direction of the auxiliary planes pass any plane through the axis of one cylinder parallel to the axis of the other. If the auxiliary planes be drawn parallel to this plane they will
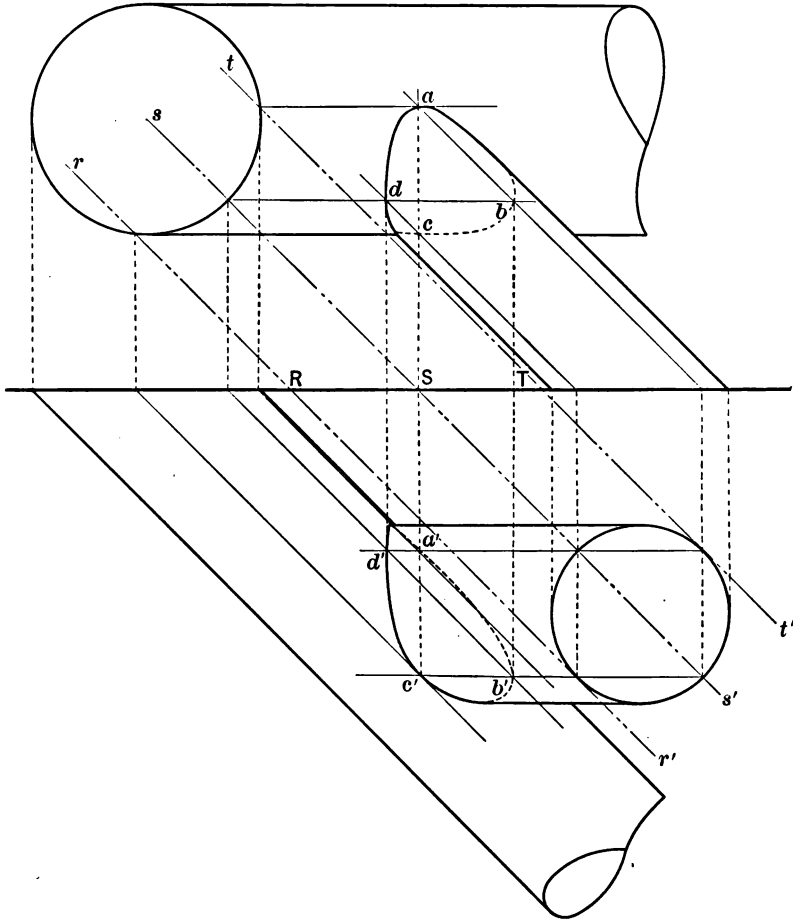
FIG. 109.

cut elements from the cylinders since they are parallel to the axis of each cylinder.

110. Something of the nature of the curve of intersection may be determined in advance. Thus, in Fig. 109, plane R is ob-

viously the first auxiliary plane which can cut elements from the smaller cylinder, and the plane T is obviously the last auxiliary plane which can cut the smaller cylinder. Therefore, since all of the elements of the smaller cylinder pierce the surface of the larger one, it becomes apparent that the smaller one completely enters the larger. On the other hand, if the plane rRr' failed to cut elements from the larger cylinder, then it would be apparent that all of the elements of the smaller cylinder did not pierce the larger, and the resulting curve would be more like the one in Fig. 108. By the use of limiting planes in this fashion the character of the required intersection may be somewhat predetermined and the problem attacked with greater appreciation.

111. The visible and invisible portions of the curve of intersection should be shown by using solid and dotted lines. To determine which points are visible it must be remembered that surfaces are not transparent, nor are they solids — unless so specified — and that when one looks into the end of a cone or a cylinder the base is not considered an obstruction to the view. A line which enters a surface is visible only up to the entering point, invisible within the surface, and visible again upon leaving the surface. The simplest method of locating visible points on a curve of intersection of two surfaces is to locate first the visible elements of the surface and of course points on these elements are also visible. It is not necessary to actually draw these elements but with a little practice and study one can readily locate the visible and invisible points by inspection.

112. **Proposition 33.** Given two cones to find their curve of intersection.

Discussion. Pass auxiliary planes through the apices of the two cones cutting elements from each. These elements being in the same plane will intersect each other in points common to both surfaces, or points on the curve of intersection.

Construction. When the bases of the given cones are in the same plane, as in Fig. 110.

Draw a line through the apices of the cones and find where it pierces H and V. These two points will be common to all the auxiliary planes. In the case given in Fig. 110 this line is par-
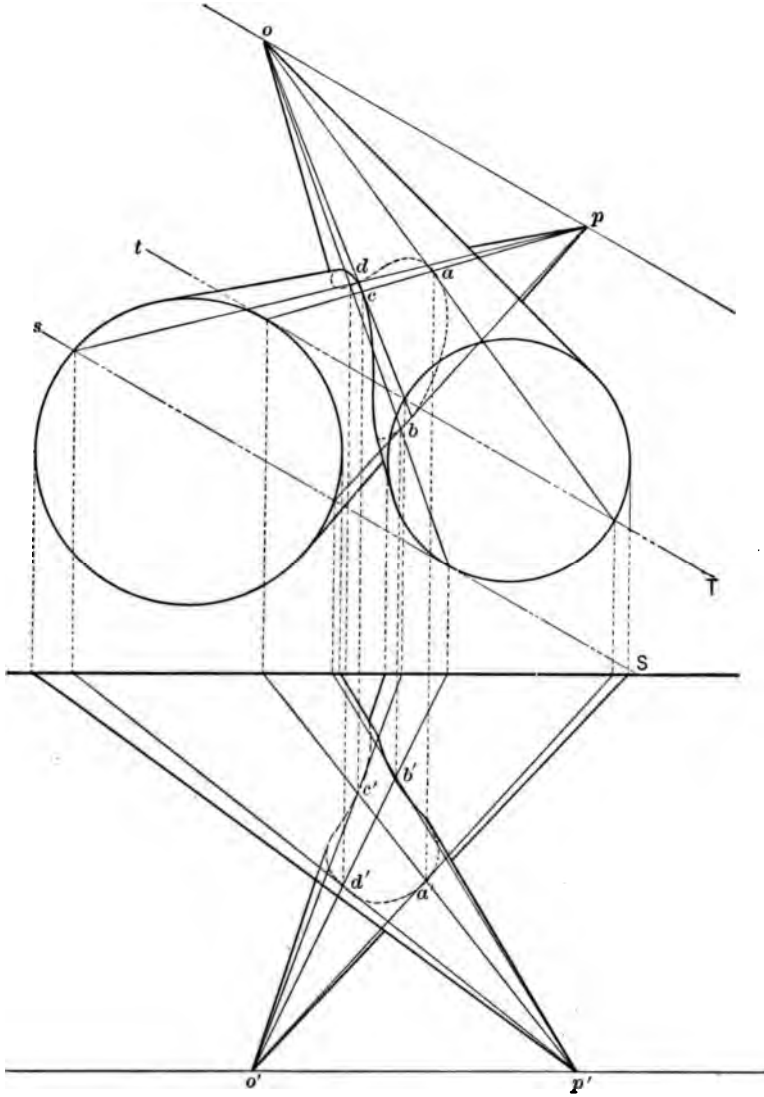
FIG. 110.

allel to H so that all the H traces of the auxiliary planes will be parallel to it. The plane S cuts one element from the right hand cone and two elements from the other; the elements intersect at B and D, two points on the curve of intersection. In like manner other points may be located but all the points common to both surfaces will lie in auxiliary planes between planes S and T.
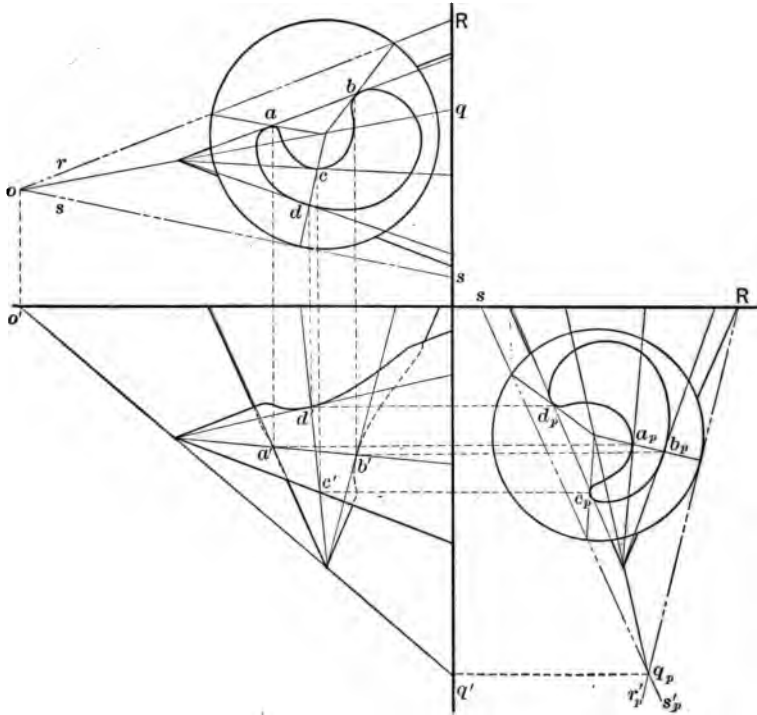
FIG. 111.

Construction. When the bases of the given cones are in different planes.

In Fig. 111 two right cones, one with its base in H and the other with its base in P, intersect. As before, the apices are joined; this line OQ is common to all the auxiliary planes and its H piercing point, O, is common, therefore, to all the H traces and its P piercing point, Q, to all the P traces. As before, locate

the limiting planes, S and R, and find the points common to both surfaces.

113. Proposition 34. Given a cone and a cylinder to find their intersection.

Discussion. Pass planes through the apex of the cone parallel to the elements of the cylinder. These planes will cut elements
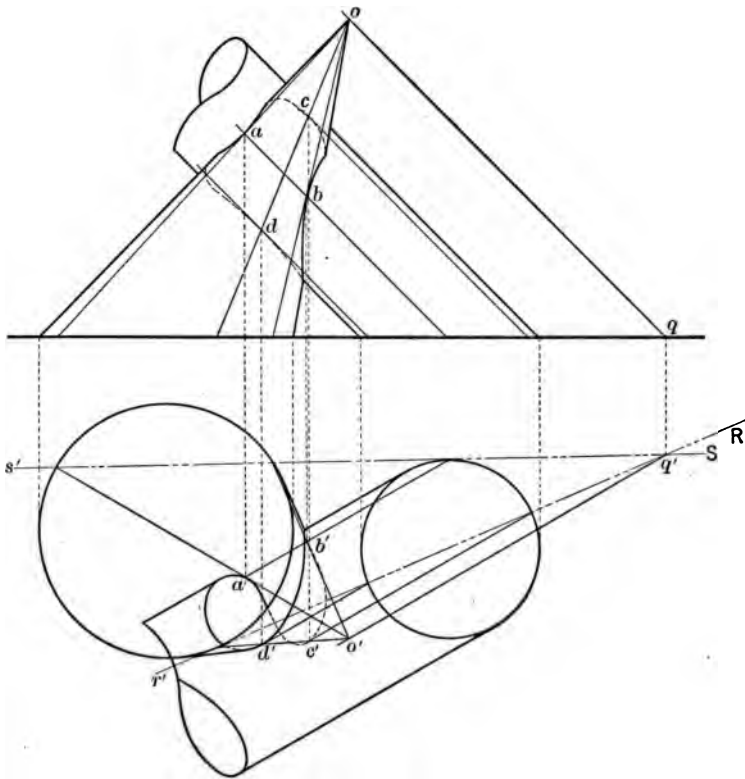
FIG. 112.

from each surface, and since these elements are in a common plane they will intersect in points common to both surfaces.

Construction. When the bases of the given surface are in the same plane.

In Fig. 112 the given cone and cylinder have circular bases in V and are oblique to both H and V. Through O, the apex of

the cone, draw a line OQ parallel to the axis of the cylinder. This line will be common to all the auxiliary planes.

The first plane whose V trace passes through Q and which cuts both the cone and cylinder is R.   Plane R cuts one element from the cone and two elements from the cylinder; these elements
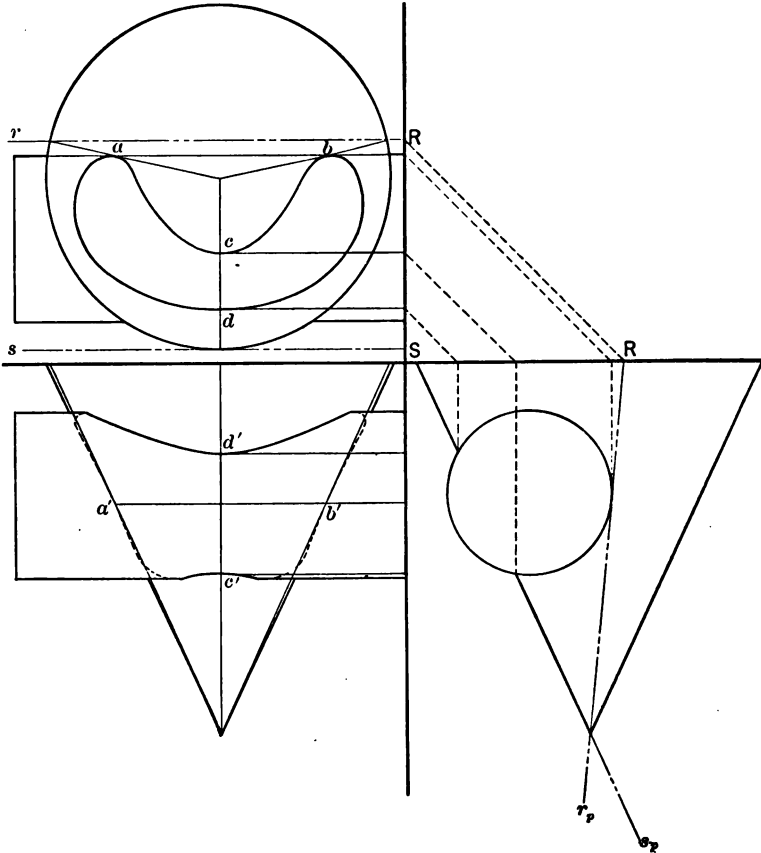
FIG. 113.

intersect at D and C which are two points on the required intersection.   The last plane which cuts both surfaces is plane S. This plane cuts only one element from the cylinder and two from the cone; these elements intersect at A and B which are two more points on the required curve.   Between these limiting

planes other auxiliary planes may be passed in sufficient number to completely locate the curve.

Construction. When the bases of the given surfaces are in different planes.

In Fig. 113 the base of the cylinder is in P and its elements are parallel to the G. L., and the base of the cone is in H and its axis is perpendicular to H. The construction will be obvious from a study of the drawing.

PROBLEMS ON THE INTERSECTION OF PLANE SURFACES

227. The axis of an hexagonal prism whose sides are 2″ wide pierces V 2″ below H, and is perpendicular to V. The axis of a square prism whose sides are 1″ wide pierces P 2″ below H and 2″ back of V, and is parallel to the G. L. The diagonals of the right section of the square prism are perpendicular and parallel to H. Draw a plan and elevation of the two prisms showing their intersection.

228. The base of a pyramid is parallel to and $\frac{1}{2}$″ below H. The base is 2″ by 3″ with its 3″ edge parallel to V 1″ back of V. The apex of the pyramid is 2″ back of V, $2\frac{1}{2}$″ below H, and in a profile plane 1″ to the right of the right edge of the base. The axis of a 1″ by 1″ prism pierces H $1\frac{11}{16}$″ to the right of the profile plane of the apex, 2″ back of V, inclines 45 degrees to H, and is parallel to V. Draw a plan and elevation showing the intersection.

229. The point M is $-2\frac{1}{2}$″; o″; -2″. The point N is $1\frac{1}{2}$″; -4″; -2″. The line MN is the axis of a prism whose base in V is a rectangle 2″ by 1″. The point O is $3\frac{1}{2}$″; -2″; o″. The point Q is o″; $-5\frac{1}{2}$″; $-3\frac{1}{2}$″. The line OQ is the axis of an hexagonal pyramid whose base in H has 1″ sides. Draw a plan and elevation showing the intersection.

230. The point A ($\frac{3}{8}$″; -4″; $-2\frac{1}{2}$″) is the apex of a pyramid whose base is CDEF. The point C is $-2\frac{11}{16}$″; $-1\frac{5}{32}$″; o. The point D is $-1\frac{7}{32}$″; -2″; o″. The point E is $-\frac{9}{16}$″; $-\frac{7}{8}$″; o″. The point F is -2″; o″; o″. The point B (-2″; $-2\frac{9}{16}$″; $-1\frac{7}{16}$″) is the apex of a second pyramid whose base is MNOP. The point M is $\frac{3}{4}$″; o″; o″. The point O is $2\frac{5}{16}$″; $-1\frac{1}{2}$″; o″. The point N is $\frac{3}{4}$″; $-1\frac{1}{2}$″; o″. The point P is $2\frac{5}{16}$″; o″; o″. Draw a plan and elevation of these two pyramids showing their intersection. Make a development of the pyramid A–CDEF with this line of intersection traced on it.

231. The axis of a 6″ square timber pierces V at o″; o″; -18″, is parallel to H, and inclines 60 degrees to V. The axis of a second timber of the same size pierces H at $+6$″; $-18'$; o″, is parallel to V, and inclines 60 degrees to H. Draw a plan and elevation of these two timbers and show also a plan and elevation of the shortest 6″ square timber which can be fitted between them.

PROBLEMS ON THE INTERSECTION OF CYLINDERS

232. The axis of a cylinder whose base in H is a 2″ circle pierces H at o; −1⅞″; o″. A second point on this axis is at 2″; −3¹³⁄₁₆″; −2″. The axis of a second cylinder whose base in H is a 1¼″ circle pierces H at 2½″; −1½″; o″. A second point on the axis of this cylinder is at 1″; −4¼″; −2⅝″. Draw a plan and elevation of these two intersecting cylinders and show the plan and elevation of their curve of intersection.

233. The axis of a cylinder whose base in H is a 2″ circle pierces H at o″; −1½″; o″. A second point on this axis is at 2″; −2¹¹⁄₁₆″; −2″. The axis of a second cylinder whose base in H is a 1½″ circle pierces H at 3½″; −1″; o″. A second point on this axis is at 2″; −2½″; −2½″. Show the plan and elevation of the curve of intersection of these two cylinders.

234. The axis of a cylinder whose base in V is a 2¾″ circle pierces V at o″; o″; −3¾″. The axis of a second cylinder whose base is a 1⅞″ circle pierces H at 1½″; −4″; o″. These two axes intersect at the point 2¼″; −2¾″; −2³⁄₁₆″. Draw a plan and elevation of the curve of intersection.

235. The axis of a cylinder whose base in V is a 1⁹⁄₁₆″ circle pierces V at o″; o″; −1¼″; a second point on this axis is at 2″; −3½″; −2¼″. The axis of a second cylinder whose base in H is a 1⅞″ circle pierces H at the point 2⅝″; −1; o″. A second point on this axis is at o″; −3⅜″; −4½″. Draw the plan and elevation of the two cylinders showing their curve of intersection.

236. The axis of a 12″ pipe lies parallel to H and 36″ below H; it inclines 60 degrees to V. The axis of a second 12″ pipe is parallel to the G. L. 24″ back of V and 60″ below H. Draw the plan and elevation of these two pipes and make the pattern for the shortest 8″ pipe which will connect them.

PROBLEMS ON THE INTERSECTION OF CONES

237. The center of the base of a cone is at o″; −3″; o″. The base is a circle 3″ in diameter. The apex of the cone is at 4½″; −5¼″; −5⅛″. The center of the base of a second cone is at 3½″; −1⅛″; o″; the base is a circle 2″ in diameter. The apex of this cone is at 2″; −5¼″; −4″. Draw a plan and elevation of the two cones showing their curve of intersection.

238. The center of the base of a cone is at o″; −3¹³⁄₁₆″; −1¾″. The base is a 3″ circle whose plane is parallel to V. The apex of this cone is at 4½″; −³⁄₁₆″; −5⅝″. The base of a second cone is at 3″; −3¹³⁄₁₆″; −2″. The base of this cone is a 2½″ circle whose plane is parallel to V. The apex is at the point o″; −³⁄₁₆″; −4¾″. Draw a plan and elevation of these two cones and show their curve of intersection.

239. The base of a cone in V has its center at the point o″; o″; −1¼″; the diameter of the base is 2½″. The apex of this cone is at 2¾″; −3¾″; −2⅜″. The base of a second cone in H has its center at 3″; −1¼″; o″; the base is

$2''$ in diameter. The apex of this cone is at $\frac{5}{8}''$; $-1\frac{7}{8}''$; $-3\frac{3}{4}''$. Draw a plan and elevation of the cones showing their curve of intersection.

240. The base of a cone in H has its center at $0''$; $-1\frac{3}{4}''$; $0''$; the diameter of the base is $2\frac{5}{8}''$. The apex of this cone is at $4\frac{1}{8}''$; $-3\frac{5}{8}''$; $-4\frac{1}{2}''$. The base of a second cone in V is at $4\frac{1}{8}''$; $0''$; $-1\frac{5}{8}''$; the base is $2\frac{1}{4}''$ in diameter. The apex of this cone is at $-\frac{3}{8}''$; $-5''$; $-2\frac{1}{2}''$. Show a plan and elevation of the two cones and find their curve of intersection.

241. The center of the base of a right cone is $2\frac{3}{8}''$ behind V, and its apex is $3\frac{3}{8}''$ below H. The cone is perpendicular to H and its base in H is $3\frac{1}{8}''$. A second right cone, which is perpendicular to V, has the center of its base $1\frac{3}{4}''$ below H in V, and its apex is $4\frac{1}{2}''$ behind V. The base of the second cone is a circle $2\frac{1}{4}''$ in diameter. Draw a plan, elevation, and end view of the two cones, and show three views of their curve of intersection.

242. The center of a $2\frac{1}{2}''$ circle whose plane is parallel to V is at $0''$; $-3''$; $-2''$. This circle is the base of a right cone whose apex is at $0''$; $-\frac{1}{2}''$; $-2''$. The center of the base of a second cone is at $2\frac{7}{16}''$; $-1\frac{5}{8}''$; $-2\frac{1}{2}''$; the apex of this cone is at $-2\frac{1}{2}''$; $-2\frac{1}{4}''$; $-\frac{3}{8}''$. The base of this second cone is a $2\frac{1}{4}''$ circle lying in the plane $1\frac{1}{2}''$; 270; 300. Show the plan and elevation of these cones and their curve of intersection.

PROBLEMS ON THE INTERSECTION OF CYLINDERS AND CONES

243. The center of the base of a cylinder lies in H at the point $0''$; $-2''$; $0''$; the base is a circle $3''$ in diameter. A second point on the axis of the cylinder is at $3''$; $-3\frac{1}{2}''$; $-1\frac{5}{8}''$. The center of the base of a cone lies in H at the point $4\frac{1}{4}''$; $-1\frac{1}{8}''$; $0''$; the diameter of this base is $2''$. The apex of this cone is at $\frac{3}{4}''$; $-4\frac{5}{8}''$; $-3\frac{1}{4}''$. Draw a plan and elevation of the cone and cylinder showing the curve of intersection.

244. The base of a cylinder has its center in H at $0''$; $-1\frac{1}{2}''$; $0''$; the diameter of the base is $2''$. A second point on the axis of this cylinder is at $2''$; $-3\frac{1}{2}''$; $-2''$. The center of the base of a cone lies in H at the point $3''$; $-1\frac{1}{2}''$; $0''$; the diameter of this base is $1\frac{3}{4}''$. The apex of the cone is at $\frac{1}{4}''$; $-3\frac{5}{8}''$; $-3\frac{5}{8}''$. Draw the plan and elevation of the cylinder and cone and show their curve of intersection.

245. The base of a cylinder has its center in V at $0''$; $0''$; $-1\frac{1}{4}''$; the diameter of this base is $1\frac{1}{2}''$. A second point on the axis of the cylinder is at $2''$; $-3\frac{7}{16}''$; $-2\frac{1}{4}''$. The center of the base of a cone lies in H at $3\frac{3}{8}''$; $-1\frac{7}{16}''$; $0''$; the diameter of the base is $2\frac{1}{2}''$. The apex of the cone is at the point $-\frac{5}{8}''$; $-2''$; $-3\frac{3}{8}''$. Draw a plan and elevation of the two surfaces and show the curve of intersection.

246. A cylinder $3''$ in diameter lies parallel to H and V with its axis $2''$ back of V and $3''$ below H. The apex of a right cone coincides with this axis of the cylinder and the projections of the axis of the cone inclines 60 de-

grees to GL.   A right section of the cone 2″ from the apex is 2″ in diameter. Draw a plan, elevation, and end view of the cone and cylinder and show their curve of intersection.

247. The axis of a right cone pierces V 1½″ above H and inclines 45 degrees to V; the apex is 4″ above H.   Between these two points the axis of the cone is 4″ long, and a right section of the cone 3″ from the apex is a circle 1½″ in diameter.   At a point 2″ from the apex the axis of the cone is cut by the axis of a right circular cylinder 2″ in diameter whose axis is parallel to H and V.   Draw three views of these two surfaces showing the curve of intersection.

CHAPTER XVI

SURFACES OF REVOLUTION

114. A Surface of Revolution is represented in a drawing by the projections of its meridian curves, or sections. These meridian curves are always taken in planes through the axis parallel to the planes of projection. Thus, in Fig. 114, the plan view of the sphere is the projection of the curve cut from the sphere by the meridian plane AB; and the elevation of the sphere is the elevation of the curve cut from the sphere by the meridian plane CD.
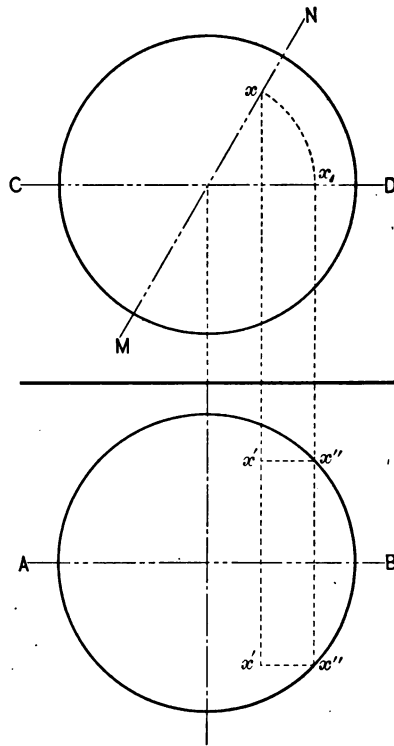
115. To assume a point on the surface of a surface of revolution one projection may be assumed, as x in Fig. 114, and the other projection found by passing a meridian plane MN through the assumed point and revolving this plane into coincidence with the meridian plane CD. In this position the point X will be projected horizontally at $x_{,}$, and vertically at x'' on the meridian curve. The plane MN may now be revolved back to its original position and the projection of point X on the surface will be found at x and x'.

FIG. 114.

It will be noted that the point projected on H at x may have two elevations and which of these is to be taken depends upon which side of the sphere the point is assumed originally.

116. In finding the intersections cut from surfaces of revolution by other surfaces the problem will be greatly simplified if the following rule is observed:

Pass the auxiliary planes through the given surface in such a direction that only the simplest and most easily located curves and elements of the surfaces are cut out.

Since surfaces of revolution have circular sections in planes perpendicular to the axes of revolution it will be obvious that, where expedient, the cutting planes should be passed in this direction.

117. Proposition 35.   Given any surface of revolution to find its intersection with any plane.

Discussion.   Pass auxiliary planes through the surface perpendicular to the axes of revolution.   These planes cut circles from the surface and lines from the given plane.   Since each circle and line lies in a common plane they will intersect in points common to the given surfaces, hence on the required intersection.

Construction.   In Fig. 115 the given surface is an ellipsoid with its axis of revolution perpendicular to H.   It is cut by the plane sSs'.   To find the required intersection pass planes through the plane and ellipsoid perpendicular to the axis or parallel to H, as R.   The plane R cuts the line MN from the plane S and the circle, whose diameter is AB, from the ellipsoid.   This line intersects the circle at X and Y, two points on the required curve. In like manner plane V cuts the circle whose diameter is CD from the surface and from the plane cuts the line OP.   These intersect at O and P, two more points on the required curve; continuing this process enough points may be found to locate the curve.   If the true size of the curve be required it may be found by revolving it into H or V about the corresponding trace of plane S.

118. Proposition 36.   Given any surface of revolution to find its intersection with any cylinder.

Discussion.   Pass auxiliary planes through the surfaces so that circles are cut from the surface of revolution and elements

from the cylinder. These elements cut the circles in points
common to both surfaces and are points of the required inter-
section.
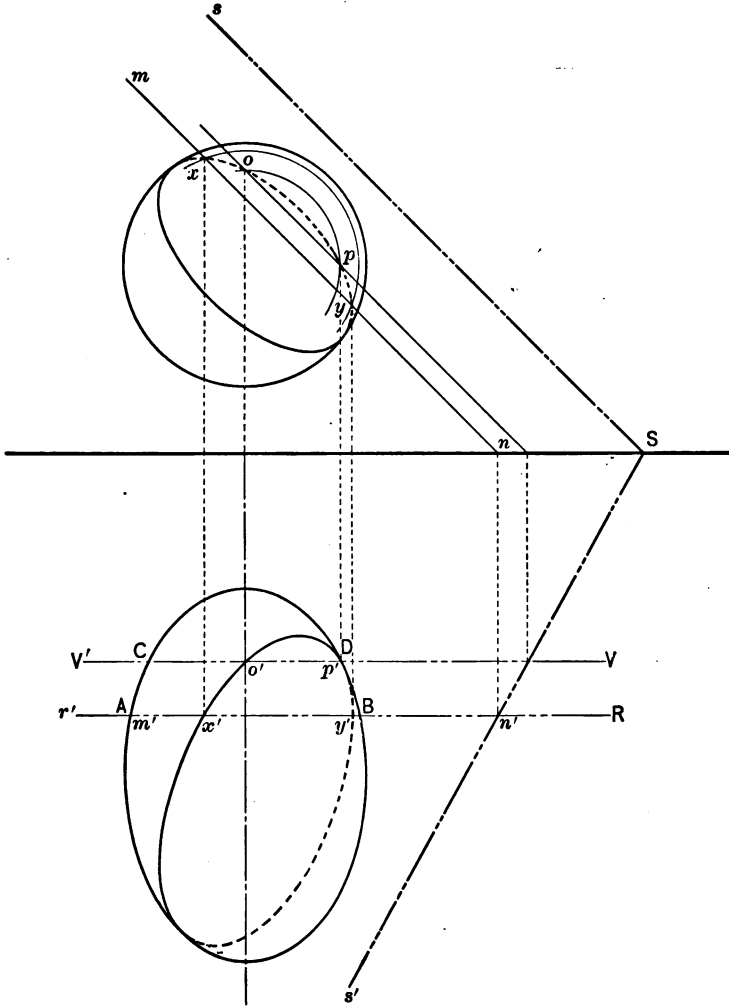
FIG. 115.

Construction. In Fig. 116 the cylinder has a circular base in
H and intersects a semicircular pipe bend (annular torus). The
auxiliary plane R cuts from the surface of the torus a circle which

is the elevation of the surface, and from the cylinder the limiting elements of the elevation.   The elements cut this circle at A and F, or two points on the required curve of intersection.   In like manner auxiliary plane S cuts a smaller circle from the torus and
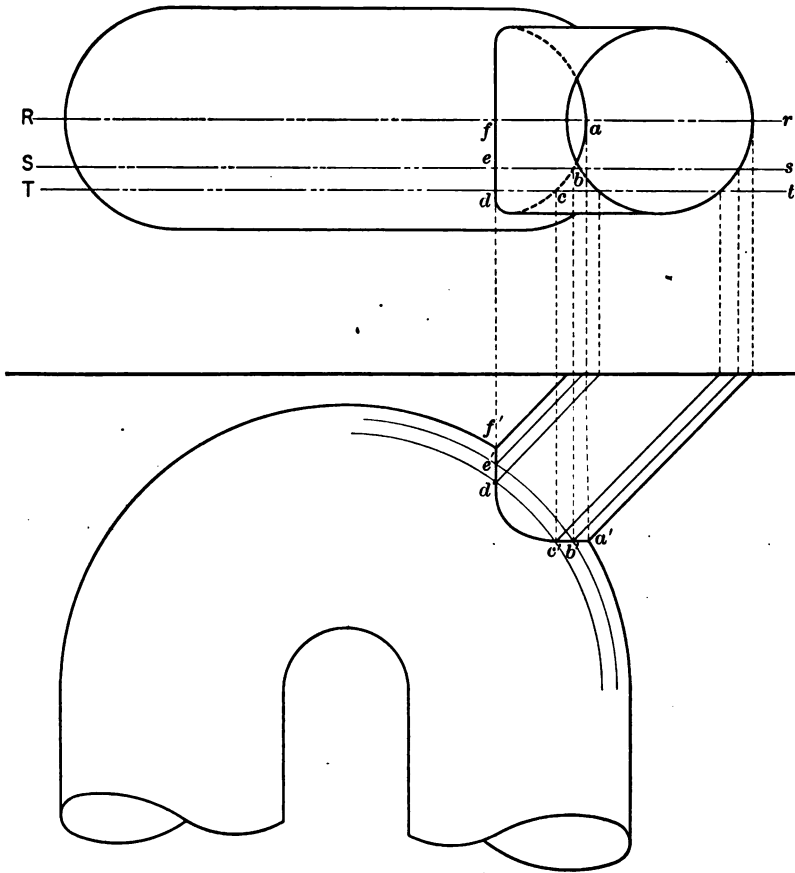
FIG. 116.

two elements from the cylinder.   These intersect at E and B, two more points on the curve.   Auxiliary plane T locates points D and C, and in this manner the entire curve may be located.

119. **Proposition 37.**   Given any surface of revolution to find its intersection with a cone.

Discussion. See Article 116.

Construction. In Fig. 117 is a cylinder of revolution with a hemispherical top, intersected by a cone with a circular base. From a study of the figure it will appear that the auxiliary planes should be passed parallel to H, for in this direction each
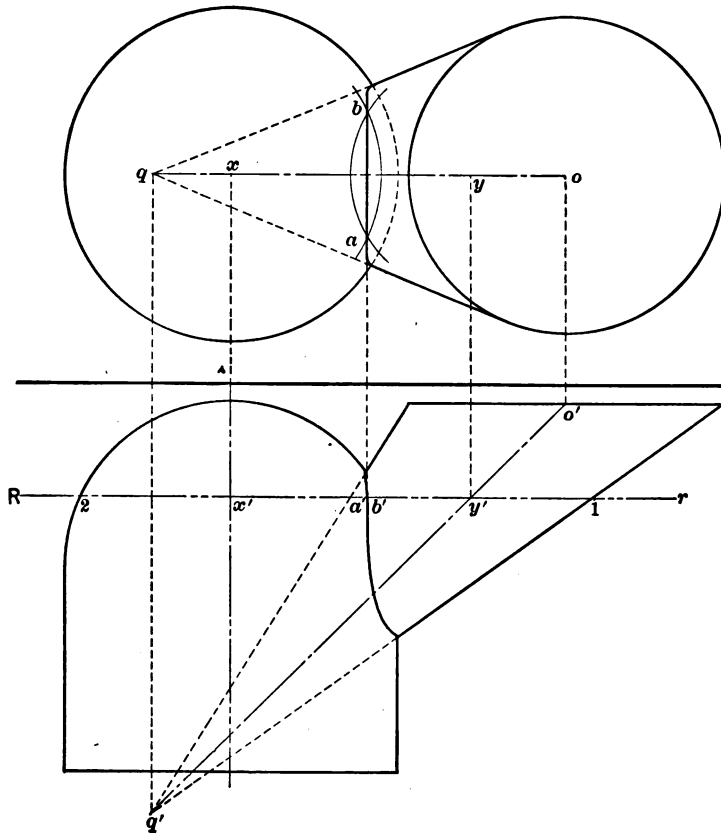
FIG. 117.

plane cuts a circle from each surface. Thus, plane R cuts from the cone a circle whose radius is $y'1$ and from the other surface a circle whose radius is $x'2$. These two circles intersect at A and B two points on the required curve. By passing other planes parallel to R the curve of intersection may be completely determined.

120. Proposition 38. Given any two surfaces of revolution whose axes intersect to find the curve of intersection of the surface.
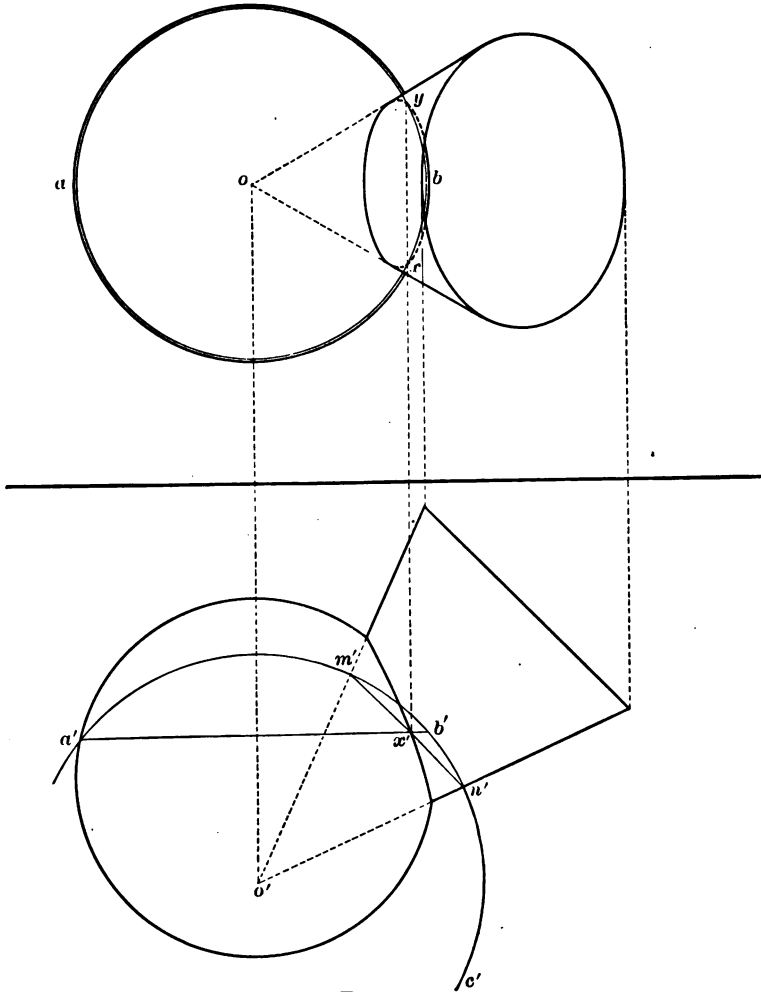
FIG. 118.

Discussion. If, with the intersection of the axes of the given surfaces as a center, auxiliary spheres be passed through the surfaces they will cut circles from each surface. These circles intersect in points on the curve of intersection.

Construction. In Fig. 118 the sphere is cut by a cone. The axes intersect at O. With this point as a center describe a sphere as ABC. This sphere cuts from the given sphere a circle vertically projected as a line at a'b' and from the cone another circle vertically projected as a line at m'n'. These two circles intersect at two points, both of which are vertically projected at x' and horizontally projected on the plan view of circle AB at x and y. By passing other spheres and proceeding in the same manner the curve of intersection may be completely determined.

121. Proposition 39. To develop any surface of revolution.

Discussion. Theoretically, surfaces of revolution are not capable of development but, practically, such surfaces are developed by methods which give approximations, which are near enough for commercial purposes. Examples of this are shown in the following developments of the sphere. In the first method the surface of the sphere is divided by meridian planes into *gores* as shown in Fig. 119. In the second method, Fig. 120, the surface of the sphere is divided into *zones* by making each zone a frustum of some cone which nearly coincides with the surface of the sphere. Often these two methods are combined in laying out the bottoms of hemispherical tanks.

FIG. 119.

Construction. Gore Method. Divide the surface into gores by meridian planes, keeping in mind the fact that the narrower the gores are made the more nearly will the development approach the true surface of a sphere. In Fig. 119 the hemisphere is divided into 16 equal gores. To lay out each gore divide the
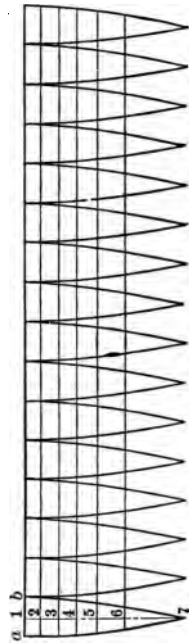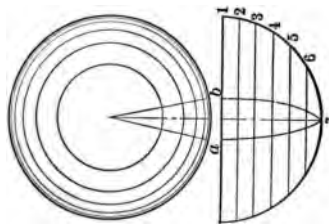
sphere as shown in the figure by horizontal planes 1–2–3–4, etc., and lay off the distances 1–2; 2–3; 3–4, etc., in the line 1–7,
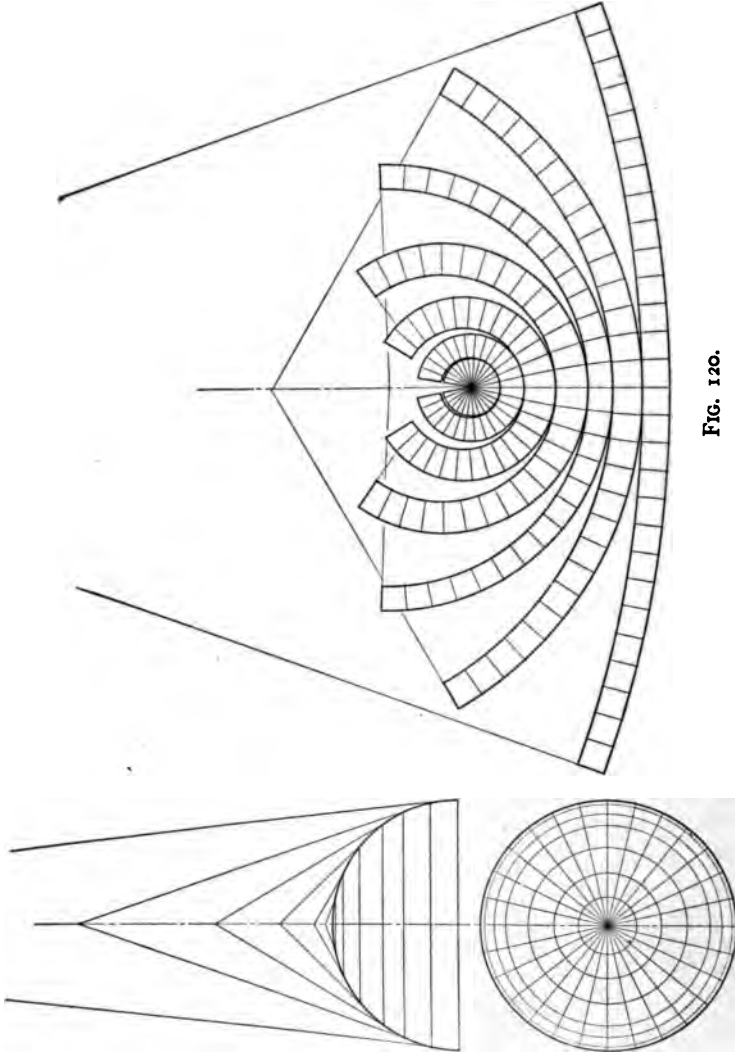
FIG. 120.

thus making the line 1–7 equal to $\frac{1}{4}$ the great circle of the sphere. Through the point 1, 2, 3, 4, etc., thus located draw horizontal lines; these will be the developed circles cut by the horizontal

planes through 1, 2, 3, 4, etc. On each of these horizontal lines lay off the distances between the meridian planes. Thus, at the point 1 lay off ab equal to the arc ab; at point 2 lay off the length of arc between the meridian planes in the plane 2. Sixteen gores, arranged as shown in the figure, developed by this method will give an approximate pattern of the hemisphere.

Construction. Zone Method. Divide the surface of the hemisphere in Fig. 120 into zones by making each zone the frustum of a cone whose apex is on the axis of the hemisphere and whose elements are tangent to the hemisphere. Develop these frusta by any convenient method of laying out a cone and arrange them as shown; the result will approximate very closely a development of a sphere.

As in the gore method, it should be remembered that the more zones used the nearer the result will approach the surface of a hemisphere.

PROBLEMS ON SURFACES OF REVOLUTION

248. An ellipsoid of revolution is generated by revolving an ellipse 2″ by 3″ about its major axis. Cut this surface by a plane oblique to both H and V and find the plan and elevation of the curve of intersection and its true size.

249. An annular torus is generated by revolving a 1″ circle about an axis lying in the plane of this circle and 1½″ from its center. Cut this torus by a plane oblique to both H and V and find the plan, elevation, and true size of the curve of intersection.

250. The center of a hemisphere is at 0″; −3″; 0″. The center of the base of a cylinder is at 5″; −3″; 0″. The radius of the hemisphere is 1½″, and the base of the cylinder is an ellipse lying in H with its 2″ major axis perpendicular to the G. L. and its 1½″ minor axis parallel to the G. L. The axis of the cylinder is parallel to V and inclines 15 degrees to H. Draw a plan and elevation of these two surfaces showing the curve of intersection.

251. A tank with a hemispherical bottom is drained by a pipe which runs off at an angle of 30 degrees to the horizontal. The tank is 72″ in diameter, the pipe is 6″ in diameter, and the axis of the pipe pierces the bottom of the tank at a point on its surface directly below the center of its hemispherical bottom. Draw a plan and elevation of the tank and pipe showing the curve of the joint between them.

252. The axis of an ellipsoid of revolution is perpendicular to H 3″ behind V. The surface is generated by the revolution of an ellipse 3″ by 2″,

the 2″ dimension being parallel to H and the upper end of the 3″ axis being 1½″ below H.   The axis of a right cone is parallel to the G. L. 3″ below H and 3″ behind V.   The apex of the cone is 2″ to the right of the axis of the ellipsoid and the center of the base lies in a profile plane 2″ to the left of the axis.   The base of the cone is a circle 2½″ in diameter lying in this profile plane.   Draw a plan and elevation of these two surfaces and show their curve of intersection.

253. In the Pierce-Arrow automobile the headlights — which are paraboloids of revolution — are set into the mud guards of the front wheels. Assuming the necessary dimensions find the shape of the hole which will have to be cut into the cylindrical mud guard to receive the lamp.

254. Make the patterns for the bottom of a tank which is a hemisphere of 42″ radius.

255. Show how to cut the tin to cover a dome which is a semi-ellipsoid in form with axes 84″ by 60″.

256. Draw three views of a U. S. Standard hexagonal nut for a 2″ bolt. The nut has a spherical chamfer.

CHAPTER XVII

WARPED SURFACES

122. A **Warped Surface** is represented by the projections of its directrices and a sufficient number of positions of its generatrix to show the character of the surface, and to indicate how the generating line moves. Thus, in Fig. 71, the projections of the directrices AB and CD are shown and a sufficient number of positions of the generatrix AX are given to indicate the character of the surface and that AX remains parallel to the H plane, which in this case is the plane director.

123. In addition to the warped surfaces which are unclassified there are three general kinds of warped surfaces:

1. Warped surfaces with two linear directrices and a plane director.

2. Helicoidal surfaces.

3. Warped surfaces with three linear directrices.

Some of these surfaces have commercial applications and such as these will be studied in detail but the majority of warped surfaces, owing to their complex nature and undevelopable character, are not used in practical work and possess only mathematical interest. The following table gives a classification of warped surfaces with the names of some of the more important ones.

CLASSIFICATION OF WARPED SURFACES

1. Warped surfaces with two linear directrices and a plane director	1. Linear directrices, both straight lines	Hyperbolic paraboloid
	2. Linear directrices, one straight and one curved	Conoid
	3. Linear directrices, both curved lines	Cylindroid

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2. Warped surfaces with a helical directrix	1. Right helicoid	Generatrix perpendicular to the axis
	2. Oblique helicoid	Generatrix oblique to axis
	3. Convolute (developable helicoid)	Not a warped surface but a single curved surface to which the helicoid approaches as a limit

3. Warped surfaces with three linear directrices	1. Linear directrices, all straight lines	Hyperboloid of one nappe. Also a surface of revolution
	2. Two linear directrices, one straight and one curved	
	3. One linear directrix straight and two curved	Warped arch Warped cone
	4. Linear directrices, all curved lines	

124. To assume a point on a warped surface one of two methods may be used. In case it is possible to draw the projection of an element through the assumed projection of the point, as in Fig. 121, this may be done and the other projection of the point may be found on the corresponding projection of the element. In Fig. 121, x′ was assumed and through it was drawn the projection p′q′ of an element of the surface passing through that point. The H projection of this element is, of course, pq, and on this projection the other projection x may be located.

In Fig. 122 this method is not applicable since through the assumed projection of the point x it is not convenient to draw the projection of the element containing it. To find x′ in this case, pass a plane R through the point perpendicular to H. This plane cuts from the warped surface the line OPQ, and obviously x′ will be found located on its projection o′p′q′.

125. Of warped surfaces having a plane director Fig. 71 is an example. This surface has two straight line directrices and is a common example of warping. This surface is called an *hyperbolic paraboloid* because one set of planes will cut hyperbolas
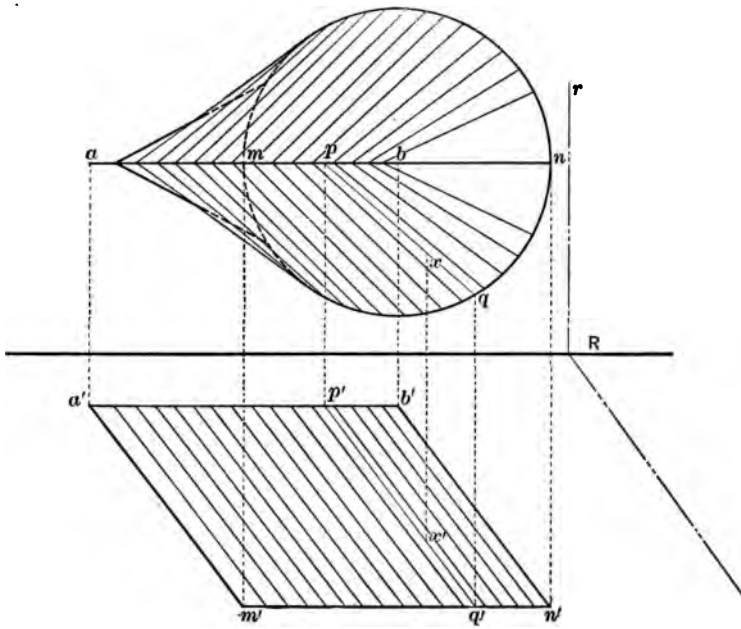
FIG. 121.
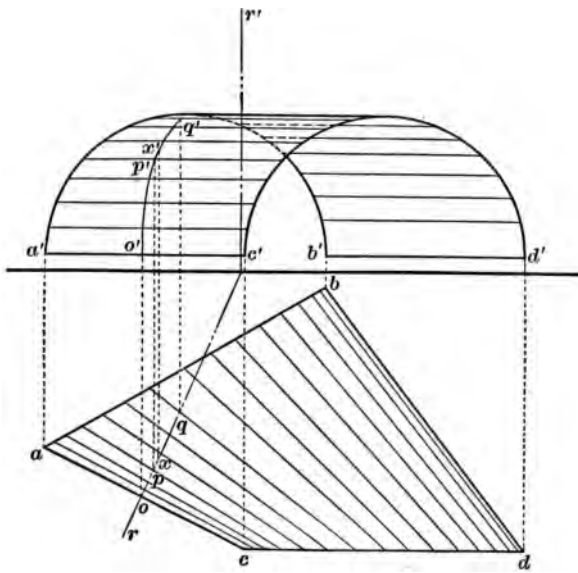
FIG. 122.

from the surface while another set perpendicular to the first set will cut parabolas. (See Fig. 123.) As practical examples of this surface the pilot on a locomotive and the bow of a ship may be cited.
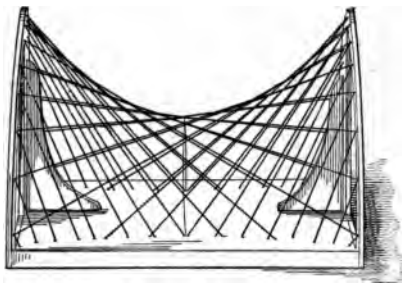
FIG. 123. — An hyperbolic paraboloid. Note that planes parallel to the base of the model will cut out hyperbolas, and that planes parallel to its ends will cut out parabolas.

If one directrix is a curved line and one remains a straight line the surface becomes a *conoid*. An example of such a surface may be found in chisels shaped from round bar stock and in the bows of ships. Fig. 121 shows an oblique conoid, and Fig. 124 shows a string model of a right conoid.

Where both directrices become curved lines the surface becomes a *cylindroid*. Fig. 122 shows such a surface and illustrates the skewed arch, one of the useful applications of the surface. The cylindroid is also made use of in buildings for connecting arched passageways at different elevations.

Many interesting applications of these and other warped surfaces may be seen in the bodies, mud guards, and other portions of modern automobiles.
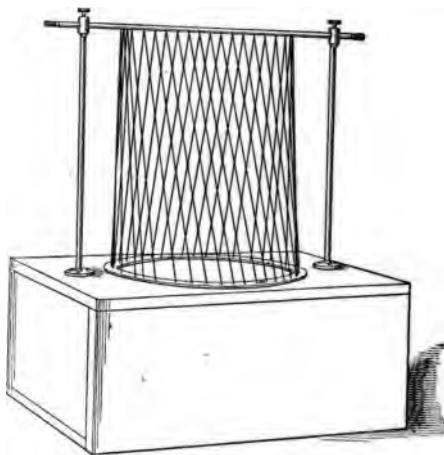
FIG. 124. — A string model of a right conoid. The base of this surface is a circle which is also the curved directrix; the bar at the top is the straight line directrix. The plane director is parallel to the ends of the box.

126. Proposition 40. Given the directrices and the plane director to draw the plan and elevation of a warped surface.

Discussion. The directrices may both be straight lines, or may both be curved lines, or one may be straight and one curved. The character of the surface will, of course, be determined by the character of these directrices as described in Article 125, but the method of drawing will be the same for all cases.

Construction. In Fig. 122 let AB and CD be the two given directrices, in this case both being curvilinear. Let the H plane be the plane director. From any point, A, on one directrix draw a line, as AC, parallel to the plane director, H. This line intersects the other given directrix at C. Therefore, since AC touches both directrices and is parallel to the plane director it will be an element of the surface. In like manner other elements may be drawn and the plan and elevation outlined. In commercial drafting only the limiting elements, or those elements which define the contour of the surface, are shown.

In Fig. 121, the oblique conoid, the method of construction is the same. In this case the curvilinear directrix is the circle MN and the rectilinear directrix is AB, and the plane director is an oblique plane rRr'. The elements must, of course, touch both MN and AB and be parallel to plane R.

127. A *helicoidal surface* is generated by a line moving uniformly about and along an axis, making a constant angle with it. If the generatrix is of definite length it is obvious that during the generation of such a surface one end of the generatrix — that touching the axis — will describe a straight line, while the other end will follow a path in space which is a curve of double curvature called a *helix*. It must be apparent that all points between these extreme points of the generatrix will also describe helices of sizes which will vary according to the position of the point on the generatrix. The surface of the helicoid, then, may be said to be made up of a series of concentric helices, and in order to study such a surface properly the nature of the helix must first be considered.

128. The helix is a space curve generated by moving a point around and along an axis at a uniform rate. The distance from the axis may be constant giving a *cylindrical helix* (Fig. 125), or it may vary uniformly giving a conical helix (Fig. 126). In

either of these cases the distance the generating point moves in
the direction of the axis during one revolution is called the *pitch*.
Practical examples of such curves may be found in the center
lines of cylindrical and conical springs (Figs. 127 and 128).
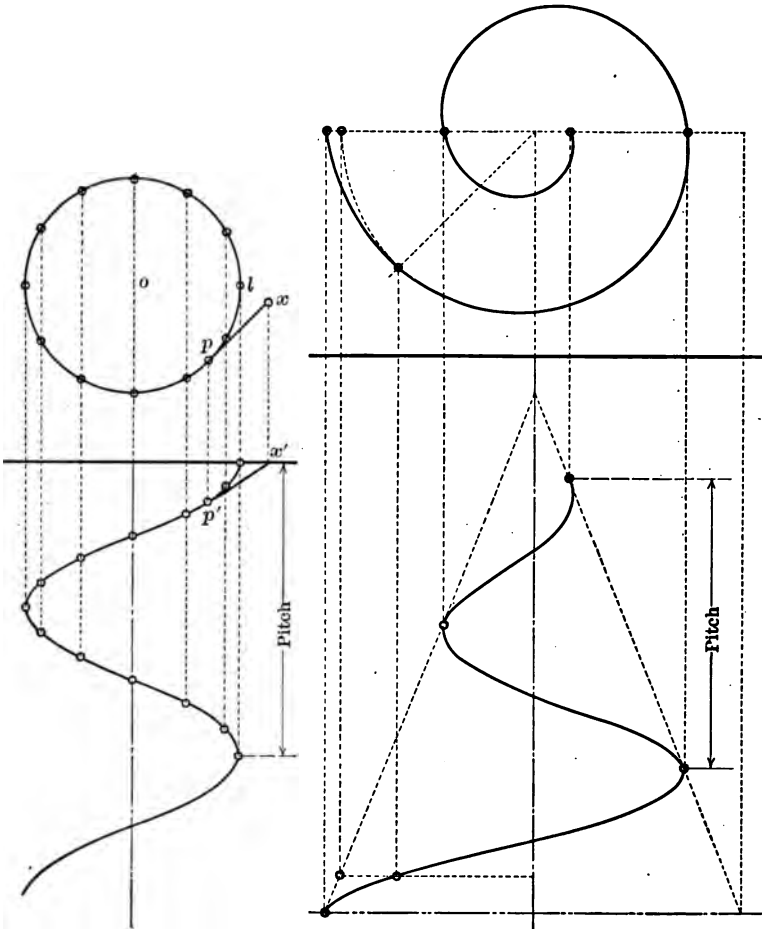
FIG. 125.   FIG. 126.

129. To construct a cylindrical helix it is necessary to know
the pitch and diameter of the cylinder on which the helix may be
wrapped.   In Fig. 129 the pitch and diameter of the cylinder are
given.   The plan view of the curve will be, of course, a circle and

may be drawn at once. To find the elevation, divide the pitch distance into any number of convenient parts, say 24, and divide the circle of the H projection into the same number of parts. Let the generating point start in H at point o. In moving from o to 1 the point makes one twenty-fourth of a complete turn
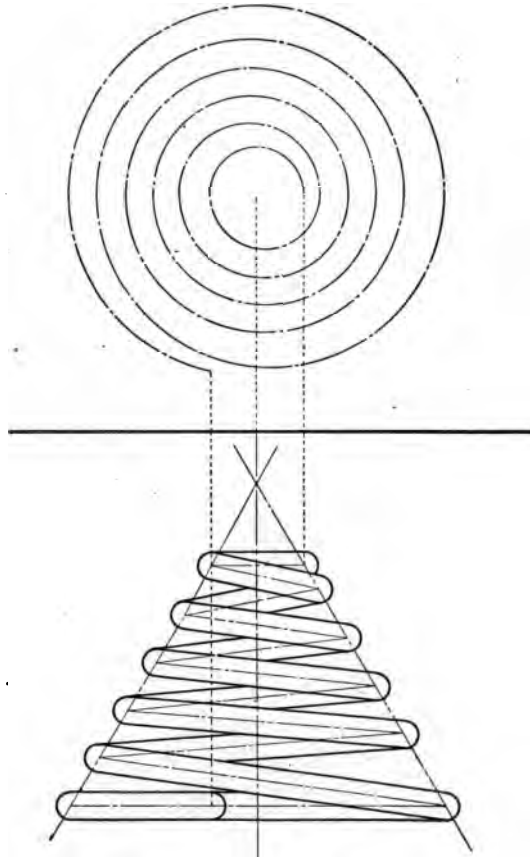
FIG. 127.

about the axis, so it therefore advances in the direction of the axis one twenty-fourth of the pitch; 1′ then is the elevation of its second position. In like manner all twenty-four positions may be located and the curve plotted.

In drawing a conical helix the same principle applies except

that it must be remembered that the H projection will be a
spiral and not a circle and that, since the helix is on the surface
of a cone, the distance of the generating point from the axis
varies directly with the distance from its first position.   Fig. 126
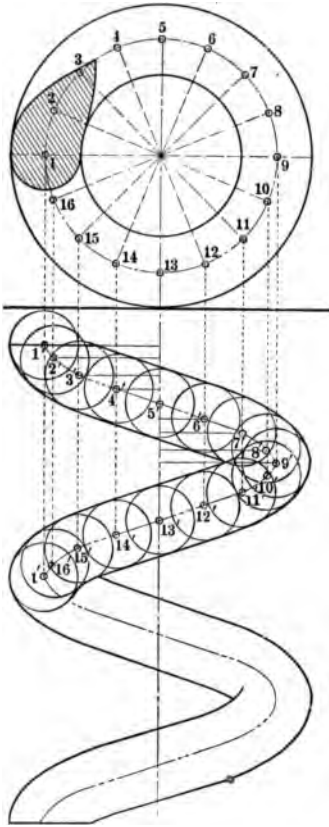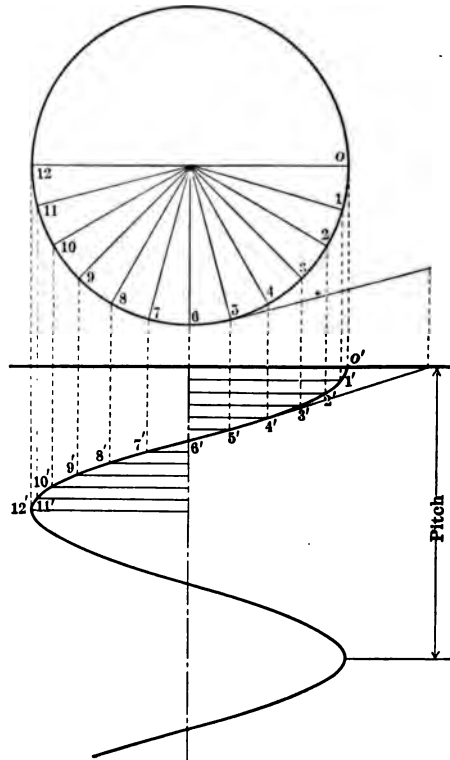
FIG. 128.   FIG. 129.

shows the construction of a conical helix and the method of
finding its projections.

130. To draw a tangent to a helix.   If in any given helix the
generating point starts from H, as in Fig. 125, it will be found
that the distance from any position of the point on the curve
back to H along the path of the helix is equal to the distance from

that same point back to H along the path of the tangent to the helix at that point. This is true because the curve and its tangent make the same angle with H, and it may be proved by wrapping a string around a cylinder in the form of a helix. Now if a portion of the string be unwound from the cylinder it will be seen that the unwrapped portion is tangent to the portion of the helix still wrapped on the cylinder, and that the end of the string always remains in the same plane. This fact is made use of in drawing a tangent to a helix. In Fig. 125 draw px perpendicular to op, and make px equal to p–1. This line will be the H projection of the tangent to the helix and x will be the point where the tangent pierces H, because px is the plan of a tangent line equal in length to the length of the helix from H to P. p′x′ then will be the elevation of the tangent and will be tangent to the elevation of the helix.

131. Proposition 41. Given the axis, the generatrix, and the pitch to draw the plan and elevation of a helicoidal surface.

Discussion. A helicoidal surface may be best represented by showing a plan and elevation of its axis, its base in H or V, and a sufficient number of elements to define the contour of the surface. For use in solving problems in connection with this surface it is convenient to have also the plan and elevation of the helix described by the end of the generating line.

Construction. In Fig. 130 let AB be the generating line in its first position. Let the axis be the line through A perpendicular to H, and let the distance that the point B rises during one revolution of AB about the axis be equal to twice the distance from 12′ to the G. L. Draw the helix described by point B according to the directions in Article 129. As the point B takes up the position on this helix indicated by the points 1, 2, 3, etc., the point A will remain in contact with the axis and will rise equal distances, as shown by $a′_1$, $a′_2$, $a′_3$, etc. By joining the successive positions of A and B the positions of the generating line as it moves about the axis may be shown. If the points where these several positions of the generating line pierce H be found, as at b, b_1, b_2, b_3, etc., the curve of the base may be located. From an inspection of the drawing it will be seen that the intersection of
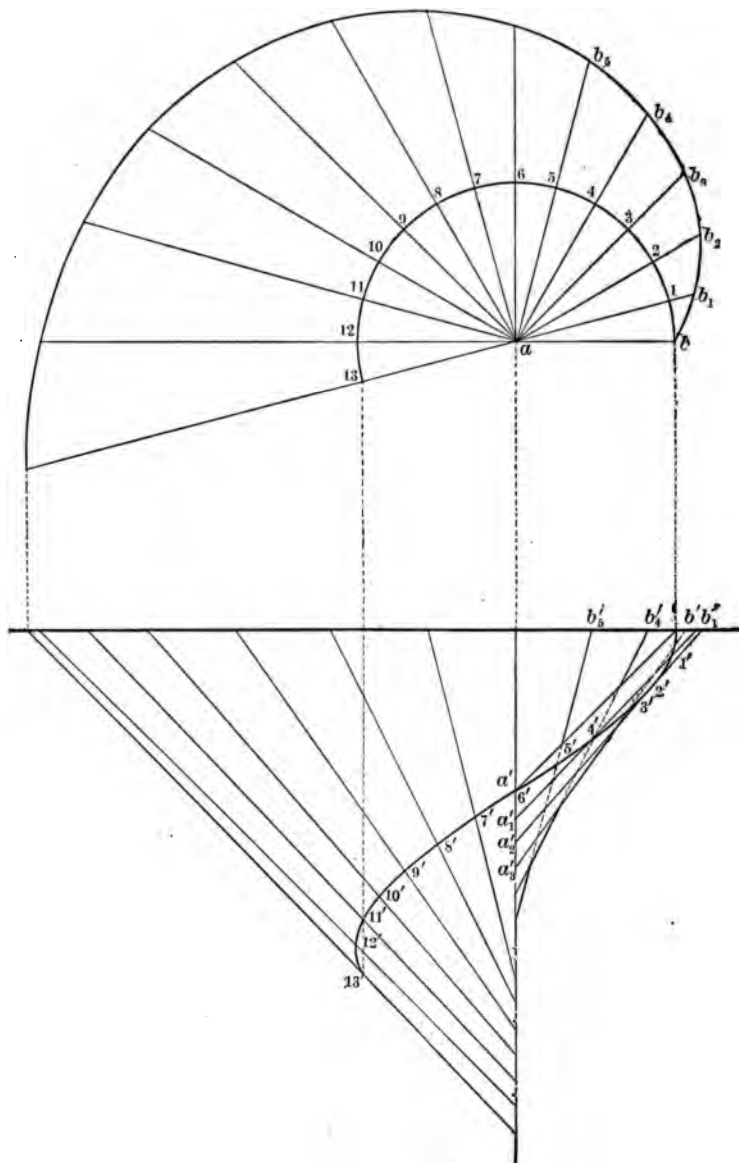
FIG. 130.

the surface with H gives a curve called the *Spiral of Archimedes.*
In the drawing only a little over one-half of one revolution of the
line AB about the axis has been shown.

132. There are two general kinds of helicoidal surfaces both
of which have many practical uses.   When the generatrix is in-
clined to the axis, as in Fig.
130, the resulting helicoid is
called an *oblique helicoid;* and
when the generatrix is per-
pendicular to the axis, as in
Fig. 134, the surface is called
a *right helicoid.* In its prac-
tical applications the heli-
coidal surface is not indefinite
in extent but is limited usually
by cylinders.   Fig. 131 shows
a plan and elevation of an
oblique helicoidal surface in
the form of a screw thread.
Fig. 132 shows a right heli-
coidal surface such as is used
in cams.   Practical examples
of helicoids may be found in
so-called "spiral" stairways,
screw conveyors, propellers,
screw threads, twist drills, etc.
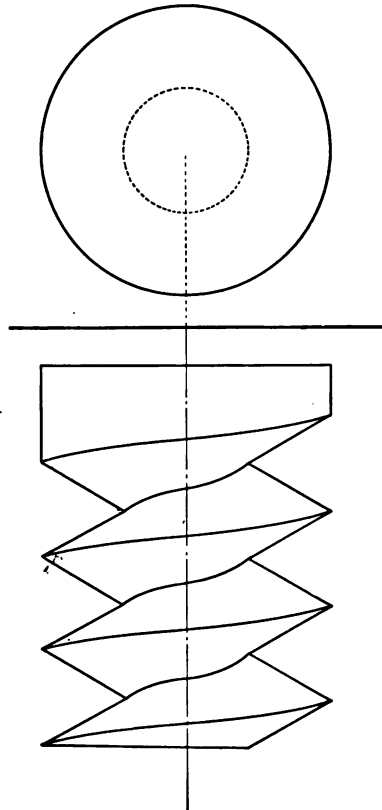
133. **Proposition 42.** Given
the axis, the pitch, and gen-
eratrix to draw the plan and
elevation of a right helicoid.

FIG. 131.

Discussion. Since nearly all right helicoids are necessarily
limited in extent for practical purposes the method by which they
are drawn may be shown by a practical illustration.   The
principle involved will be, of course, the same for any similar
surface.

Construction. Let OP be the given axis in Fig. 133, and let
O1 be the given generatrix which advances the given pitch dis-

tance during one revolution.   The point 1 in revolving about the
axis describes a circle whose plan view is shown at 1——13——1,
and moves up or down the given pitch distance.   Therefore,
divide the circle into 24 equal parts and the pitch distance into
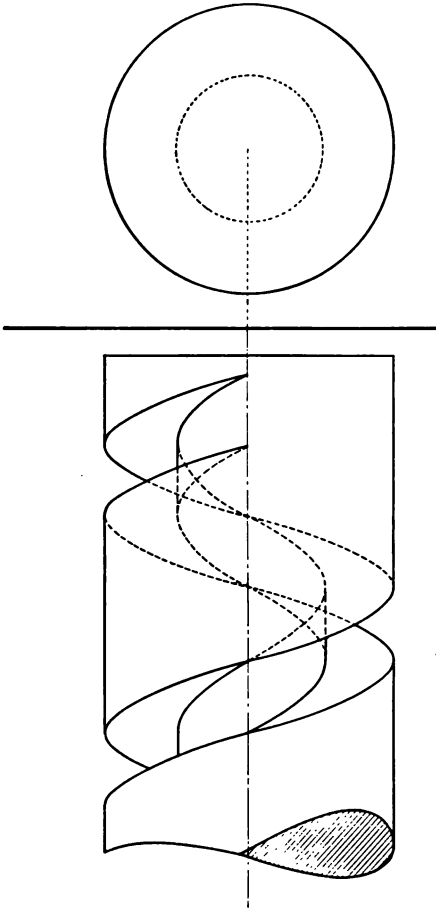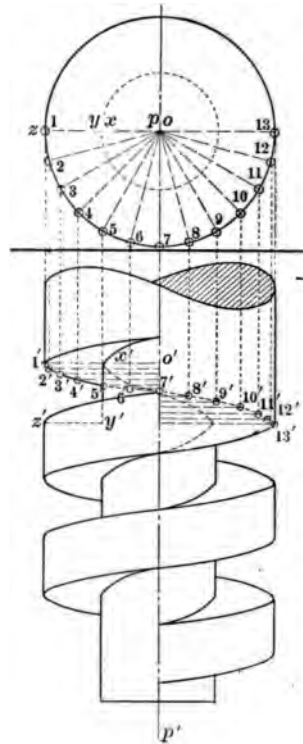
FIG. 132.   FIG. 133.

the same number of parts.   As the point moves from 1 it will
also advance so that it will take up the position 2, 3, 4, etc.,
thus describing a helix; and at the same time O moves along
the axis so that the motion of the line O1 generates a right
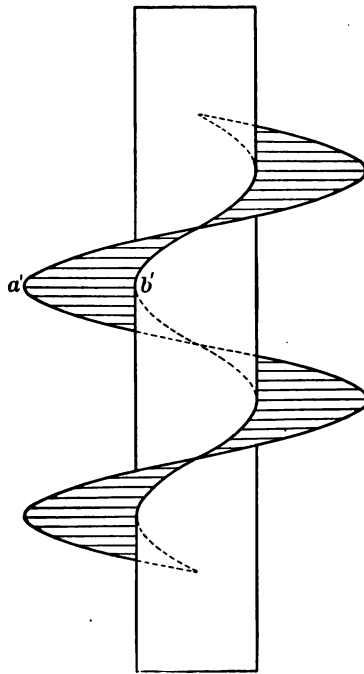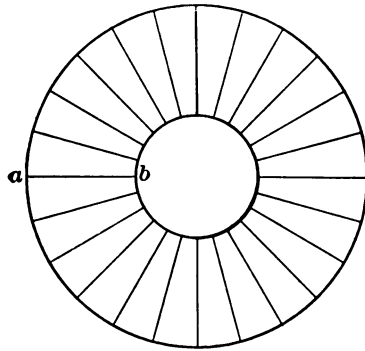
FIG. 134.

helicoid. In the given figure the square 1XYZ, of which the line O1 is one side, has been revolved, thus cutting in the cylinder a square thread, whose faces are right helicoids.

Fig. 134 shows a right helicoidal surface in the form usually

used for conveyors. The flights are right helicoidal surfaces and are mounted on a shaft. When the conveyor is enclosed in a hollow cylinder and revolved it is capable of delivering material along the length of the enclosing casing.

134. When the axis of a helicoid becomes a cylinder the surface becomes a convolute, and for this reason the convolute, which is a single curved surface, is often called the *developable helicoid.* The helicoid is the limiting form of the convolute, for as the cylinder, about which the directing helix is wrapped, approaches a straight line so does the convolute surface approach the helicoid as a limit. A helical convolute, then, is generated by a line moving tangent to a given helix. (See Fig. 135.)
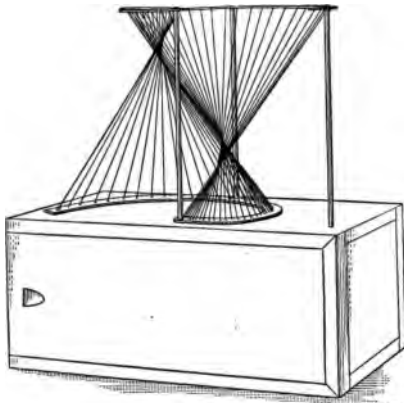
FIG. 135. — A string model of a developable helicoid. The strings are each tangent to the helix of which they form the envelope, and which may be seen in the illustration.

135. Proposition 43. Given the directing helix to draw the plan and elevation of a convolute surface.

Discussion. A convolute is usually represented by its base and the plan and elevation of the directing helix, and a number of elements. Since the surface is generated by a line moving tangent to the directing helix, it becomes necessary to draw a number of tangents to the helix and find where they pierce H or V to get the base.

Construction. In Fig. 136, MN is the axis of the given helix ABCD. By Article 130 draw a number of tangents to the helix and find where these tangents pierce H at 1, 2, 3, 4, etc. The curve through these points will be the base in H.

136. In Fig. 137 is shown the plan and elevation of a conveyor, whose flights are made developable helicoids. To draw the plan and elevation of such a surface the diameter and pitch of the smaller helix — the directrix — must be known, and also

the length of the generatrix or the outside diameter of the con-
veyor flights.

Given these data the drawing is made as follows:

Tangent to the directing helix at B draw AB. A is the point
where the line, which is a position of the generatrix parallel to
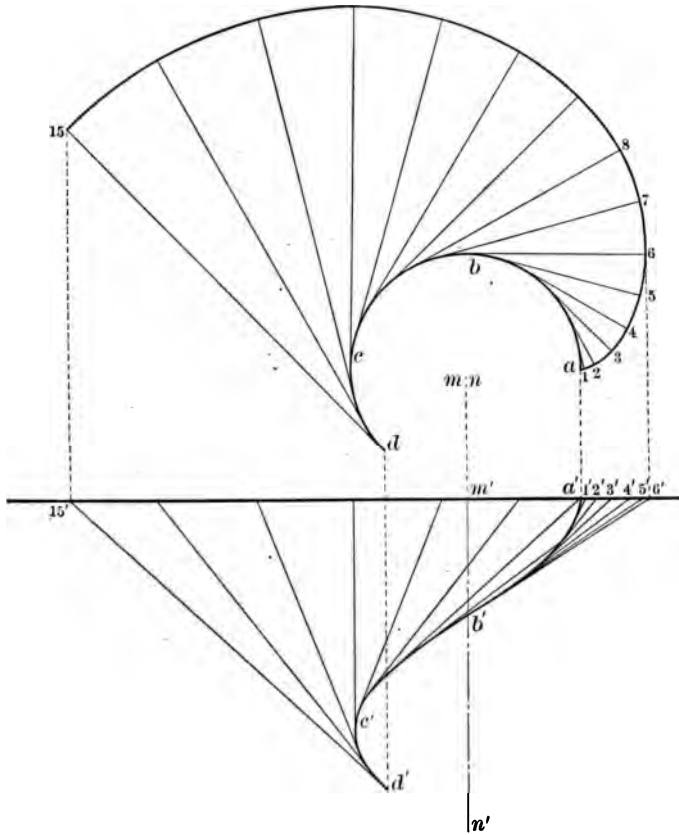
FIG. 136.

V, pierces the limiting outside cylinder; A, then, is a point on
the limiting outside helix. With A as one point construct a helix
with a pitch equal to the directing helix on the limiting outside
cylinder. If lines be drawn by connecting the points in order,
as A and B, etc., they will be elements of the surface. It
should be noted that with these same given data another con-

volute may be constructed with AC as the generating line, in which case the surface will slope in the opposite way to the surface shown.

137. To develop a convolute. One flight, or turn, of a convolute such as shown in Fig. 137 becomes a portion of a flat

DEVELOPMENT OF ONE FLIGHT
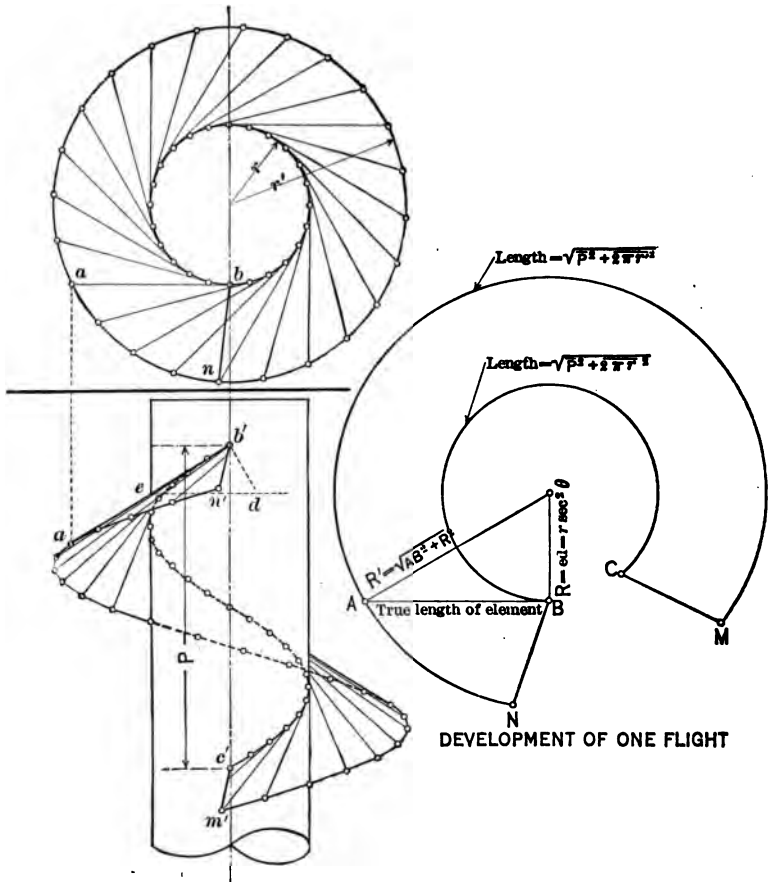
FIG. 137.

ring when developed. The radius R, of the inner circle of the ring, is equal to the line ed since the tangents have a constant slope. This length is obtained by drawing a perpendicular b'd to the tangent a'b', and from the point e where a'b' cuts the elevation of the limiting element of the inner cylinder a hori-

zontal line ed.   The length of this line ed is the graphical solu-
tion of the equation r sec$^2 \cdot \theta$, in which r is the radius of the inner
cylinder and θ is the angle the tangent AB makes with H.   The
radius R$'$, of the outer circle of the developed ring, is found by
drawing AB tangent to the developed inner helix.   Since AB is
the true length of an element the point A will therefore be a
point on the circumference of the outer circle, and through this
point this circle may be drawn.   The length of this edge of the
ring will be the developed length of the outer helix and may be
found by solving the equation $\sqrt{P^2 + 2\pi r'^2}$, in which P is the
pitch of the helix and r$'$ is the radius of the plan view of the helix.
This length may be found graphically by laying off a right tri-
angle whose base is equal to the circumference of the plan view of
the helix and whose altitude is equal to the pitch.   The hypothe-
nuse of this triangle is equal in length to the helix for one turn.
The length of the inner edge of the ring may be found in a similar
manner.

138. The helix is used extensively in the generation of other
surfaces of commercial importance.   Of these, screw threads
and the so-called " spiral springs " are the best examples.   Coiled
springs made up of rectangular stock are right helicoids while
those made up of round stock, such as the serpentine shown in
Fig. 128, are surfaces of double curvature.

139. Of warped surfaces of the third class, — those having
three linear directrices, — comparatively few are used in practical
work.   These directrices may all be curved lines, or one may be
curved and two straight, or two may be curved and one straight,
or all may be straight.

The peculiarity of this last surface — which is called the hyper-
boloid of revolution of one nappe — is that it is the only sur-
face of revolution which is warped.   It may be generated by
revolving a straight line about another not in the same plane
with it.   This surface is a surface of revolution, and, since it is
possible to have the generatrix slope two ways and generate the
same surface, the surface is doubly ruled.   (See Fig. 138.)   It is
difficult to imagine this surface as being also generated by a line
moving so as to touch three other straight lines but that it can

be done can be proved. If any three positions of the generating line as it revolves about the axis be taken as directrices, and a position of the generatrix of the opposite slope be moved so as to touch them, it will follow the surface.

The surface is used to some extent in transmitting motion from one shaft to another not in the same plane with it. The principle by which this is accomplished is shown in Fig. 139.

If the line AB be revolved about each shaft it will generate two hyperboloids having a common element, or AB. Naturally, then, when these hyperboloids roll on their respective axes they will constantly be in contact and therefore one may be made to drive the other.

140. Another warped surface of this class having two curved directrices and one straight directrix is sometimes met with in arches. If the two curves are semicircular, as in Fig. 140, and lie in parallel planes, and if the straight line directrix MN lies in a plane perpendicular to the planes of the curved directrices through their centers the surface becomes a *cow's horn arch*.

FIG. 138.—A model of an hyperboloid of one nappe showing by the position of the strings how the surface may be doubly ruled.

A similar surface of this same general character is the *warped cone*, shown in Fig. 141. In this surface the curvilinear directrices do not necessarily lie in parallel planes nor is the rectilinear directrix in a plane perpendicular to them. A common application of this surface is shown in Fig. 141, where the warped cone is used as a hood. The method of construction will be obvious from an inspection of the figure.

141. Proposition 44. To develop a warped surface.

Discussion. While warped surfaces may not be truly developed, patterns for some warped surfaces may be laid out which,
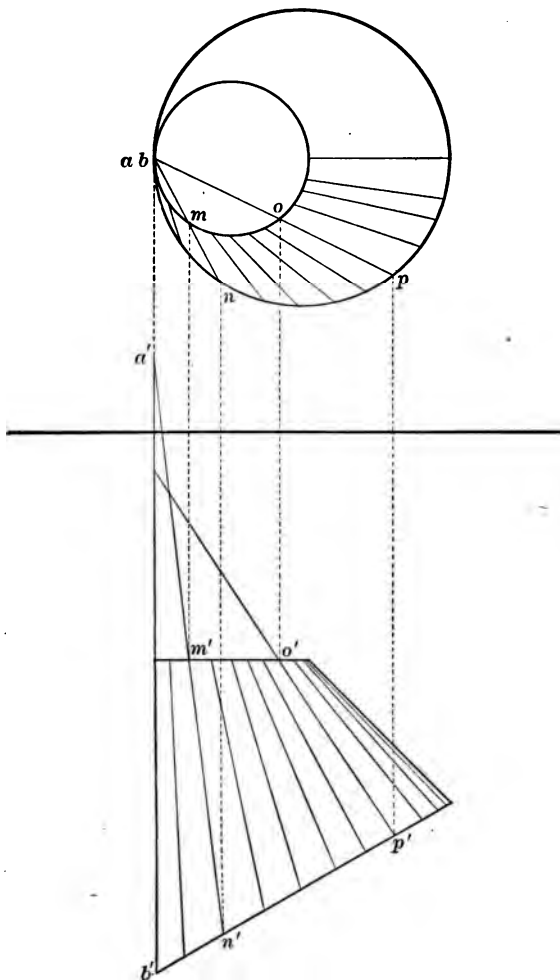
FIG. 141.

when formed, approximate the original surface closely enough for commercial purposes. An example of this is shown in the development of the warped cone in Fig. 142.
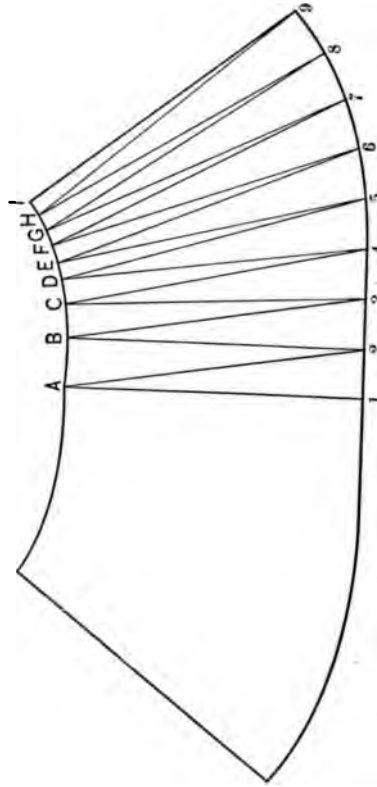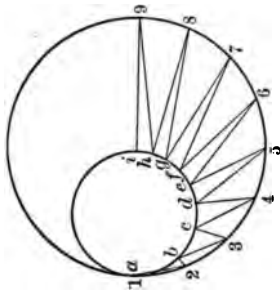
FIG. 142.
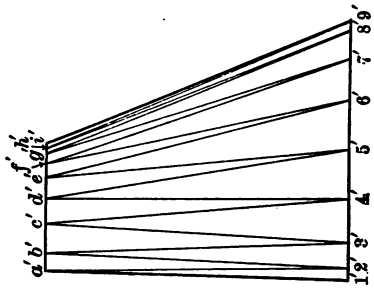
Construction. Make the development by the method of triangulation as explained in Article 107.

142. There are many other forms of warped surfaces than those which are here considered; but as they are seldom encountered in practical work and possess, therefore, little interest to the engineer or architect, it has been thought well to omit them. The principles by which warped surfaces in general are represented and by which problems relating to such surfaces are solved have been discussed in detail, and no peculiar difficulties should be encountered in solving problems relating to the more obscure surfaces omitted from consideration here.

PROBLEMS ON WARPED SURFACES

257. The plane T is $0''$; 90; 45. The point O lies in plane T at $1\frac{1}{4}''$; $1\frac{1}{4}''$; $0''$ and is the center of a $2''$ circle also in the plane T. The plane S is $5''$; 90; 150. The point Q lies in the H trace of plane S in the profile plane $3\frac{1}{2}''$; 90; 90 and is the center of a $3''$ circle lying in plane S. With these two circles as directrices and the V plane as plane director draw a plan and elevation of a cylindroid.

258. The plane S is $0''$; 90; 30. The point O is in the H trace of plane S and in the profile plane $2\frac{1}{4}''$; 90; 90. This point is the center of a $2\frac{1}{2}''$ circle lying in plane S. The line MN is perpendicular to V at the point $4\frac{1}{2}''$; $0''$; $-1\frac{1}{4}''$. With the circle and line as directrices and the V plane as plane director draw a plan, elevation, and end view of a conoid.

259. The point A is $0''$; $-3''$; $0''$. The point B is $3''$; $0''$; $-3''$. The point C is $4''$; $0''$; $-4\frac{1}{2}''$. The point D is $5\frac{1}{2}''$; $-3\frac{1}{2}''$; $0''$. With AB and CD as directrices, and with the H plane as plane director draw the plan, elevation, and end view of the hyperbolic paraboloid.

260. A cylinder $2\frac{1}{2}''$ in diameter has cut on it a single square thread whose pitch is $\frac{3}{4}''$. Draw a plan and elevation.

261. A $4''$ screw has 3 standard V-threads per inch. Draw a plan and elevation of the screw.

262. A bundle conveyor passes from the top floor of a building to the wrapping counter in the basement. The distance between floors is $10'$ and in passing from one floor to the floor below the bundle makes 3 turns. The conveyor is in the form of a right helicoid $5'$ in diameter and mounted on a $2'$ shaft. Draw a plan and elevation of the conveyor.

263. Draw the plan and elevation of a screw conveyor $14''$ in diameter mounted on a $3''$ shaft. The conveyor is capable of delivering 7500 cubic feet per hour when revolving 100 revolutions per minute.

264. Draw the plan and elevation of a helical spring made of ¼″ square stock. Inside diameter of spring 2½″; length 6″; pitch 1″.

265. Draw the plan and elevation of the spring in Problem 264 made up of ⅜″ round stock.

266. A conical spring is 6″ in diameter at its base and 2″ in diameter at its top. It is made up of 4 turns of ¼″ square stock. Draw the plan and elevation of the spring.

267. Draw a plan and elevation and give the length of the stock required to make the spring in Problem 266 out of ⅜″ round stock.

268. Draw the plan and elevation of a convolute conveyor whose diameter is 18″, whose pitch is 16″, and whose shaft is 6″ in diameter.

269. Draw a plan and elevation of a wood screw which is formed by two convolute surfaces. Outside diameter 4″, diameter at the root of the thread 2″, pitch 2″. ,

270. A railway which runs due north and south crosses over a street which runs north 75 degrees east on a cow's horn arch. The openings in the arch are circles 40′ in diameter lying in parallel planes 80′ apart. Draw a plan, elevation and end view of the arch.

271. A reducing hood whose opening is a circle 36″ in diameter and whose outlet is a circle 8″ in diameter is fastened to a wall so that the circular openings are tangent to the wall. These tangent points are in a line which is vertical and which is the straight line directrix of the surface. The length of the hood along the wall is 42″, the plane of the opening inclines 45 degrees to the wall, and the plane of the outlet is perpendicular to the wall. Draw a plan and elevation of the hood and make a pattern for it.

272. A reducing hood has two circular openings whose centers are in a line perpendicular to H, and 36″ apart. The outlet of the hood is a circle 18″ in diameter whose plane is perpendicular to this line; and the inlet is a circle whose diameter is 48″ and whose plane inclines 30 degrees to this line. Draw a plan and elevation of this hood and make a pattern for it.

273. Make a pattern for a convolute conveyor flight whose diameter is 12″, pitch 8″, diameter of shaft 3″.

274. Two shafts, one vertical and one inclined 45 degrees in a vertical plane, are 2″ in diameter, and their center lines are 7″ apart. Draw the plan, elevation, and end view of two hyperboloids of revolution which will operate on these shafts in the ratio of 4 to 3.

CHAPTER XVIII

TANGENT PLANES AND LINES

143. A plane is tangent to a ruled surface when it contains one and only one element of the surface. The element common to the tangent plane and the surface is called the element of contact.

144. A plane is tangent to a double curved surface when it contains one and only one point of the surface. The point common to the tangent plane and the surface is called the point of contact, or the point of tangency.

145. A line is tangent to a curve when it contains one and only one point of the curve. The point common to the curve and the line is called the point of tangency.

146. A plane which is tangent to a surface contains all of the lines tangent to curves of the surface at the point where these curves touch the tangent plane. In a single curved surface such as a cone, for example, every line tangent to a curve on the surface of the cone will lie in the plane tangent to the cone along the element which passes through the point of tangency. Likewise in a double curved surface such as a sphere every circle which passes through the point of contact will have a tangent line which will lie in the plane tangent to the sphere at that point.

147. Proposition 45. Given any single curved surface to find the plane tangent to it at a given point on the surface.

Discussion. If an element of the surface be drawn through the given point it will be the element of contact. Since this line lies in the tangent plane it will pierce H and V in the corresponding traces of that plane. The tangent plane will also contain any tangent to any curve of the surface at the point where this curve passes through the element of contact. Since the

base of the given surface is such a curve it will be obvious that the trace of the required plane will be tangent to that base.

Construction. Let the given surface in Fig. 143 be a cylinder with its base in V and the given point be O. Draw the element OX through O. This will be the element of contact and since it pierces V at X this point will be on the V trace of the required
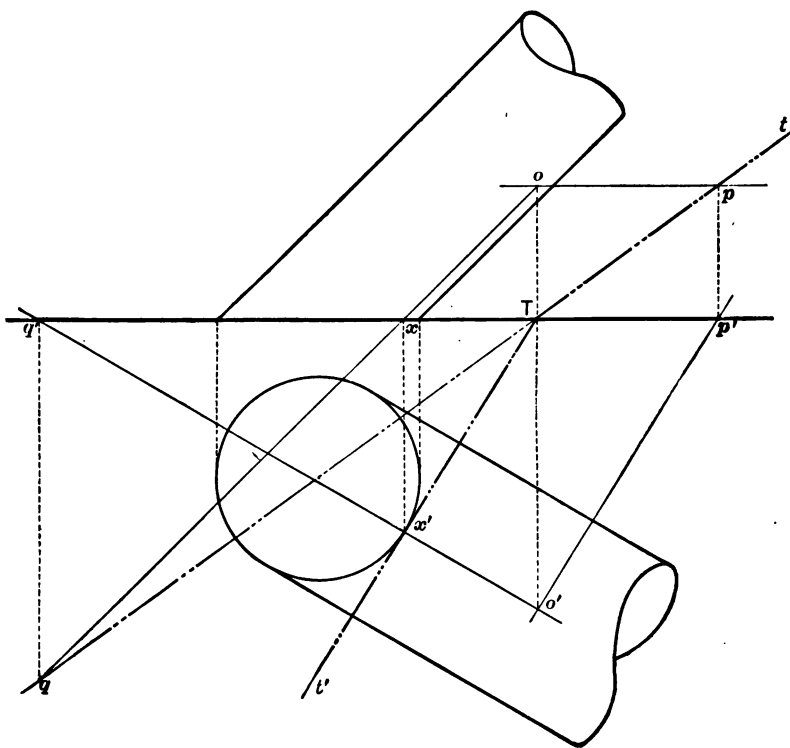
FIG. 143.

tangent plane. Through X draw Tt′ tangent to the base of the cylinder. This will be the V trace of the required tangent plane. The H trace may be found by locating Q, the point where the element of contact pierces H, and through this point and T drawing the H trace Tt. Since Q is very often remote or inaccessible a more convenient way of finding the H trace is to draw through O a line parallel to the V trace, as the line OP.

Since OP is parallel to a line of the plane through a point in the plane it will itself lie in the required plane and will, therefore, pierce H in a point on the required H trace. P is this point and Tt may be drawn through P and T.

148. Proposition 46. Given any single curved surface to find a plane tangent to it through any given point not on its surface.

Discussion. If through the given point without the surface a line be drawn parallel to the element of contact, or through some point of the element of contact, this line will lie in the tangent plane and will pierce H and V in the corresponding traces of the required plane. Through these piercing points the traces may then be drawn as in the previous proposition.

Construction. When the given surface is a cylinder. Draw a line through the given point parallel to the elements of the cylinder. This line will lie in the tangent plane and will pierce H and V in the required H and V traces of the tangent plane. With these points known the traces may be located as in Proposition 45. Let the drawing be made in accordance with these suggestions.

Construction. When the given surface is a cone as in Fig. 144. Let O be the given point. Join O with the apex Q. This line OQ must lie in the required tangent plane since it contains O, the given point, and Q, the apex through which all tangent planes must pass. OQ pierces H at P and pierces V at M. Through P draw Tt tangent to the base of the cone; this will be the H trace of the required tangent plane. Through Q draw a line QN parallel to Tt; this will be a line in the required plane. It pierces V at N. Through N and M draw t'T, the V trace of the required plane. If the traces meet at the ground line the work is checked.

Construction. When the given surface is a convolute. Through the given point draw a line making the same angle with H, or V, as the elements of the convolute make with H, or V. Let this line generate a right cone whose base will be a circle lying in the plane of the base of the given convolute. If now a plane be passed tangent to this cone it will contain the given point, and if this same plane be made tangent to the con-

volute it will be the required ·plane.   To fit these conditions one
trace of the plane must be tangent both to the base of the cone
and to the base of the convolute.   The other trace may be lo-
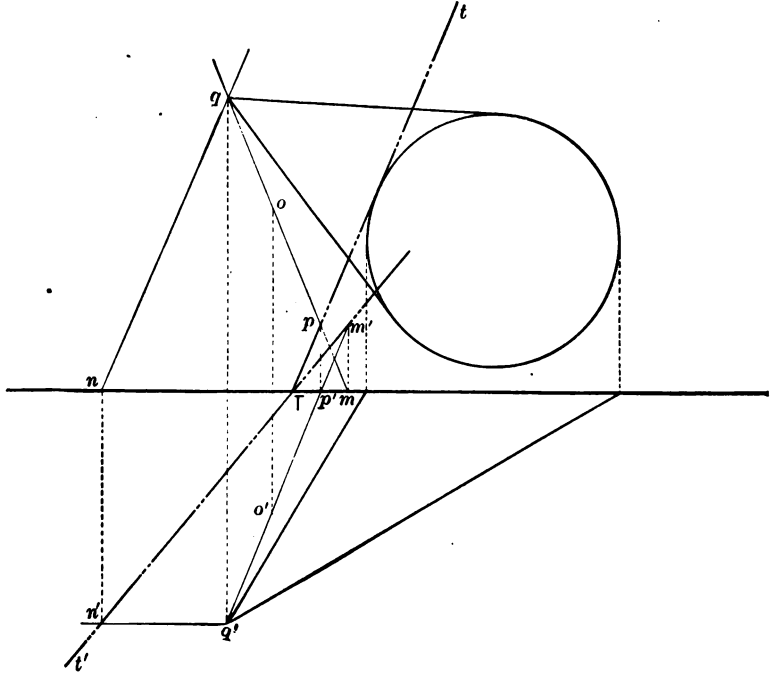
FIG. 144.

cated as in the foregoing propositions.   Let the construction
be made in accordance with these suggestions.

149. **Proposition 47.**   Given any single curved surface to
find a plane tangent to it which is parallel to any given line.

Discussion.   Since the plane is to be tangent to the given
surface it must, therefore, contain an element of the surface.
Also since it is to be parallel to a given line it must contain a
line parallel to the given line.   Since the two lines may be made
to intersect they will determine the tangent plane as indicated
in the following constructions.

Construction.   When the given surface is a cylinder.   Through
any point on the given line draw a line parallel to the elements

of the cylinder. The plane of these two lines will be parallel to
the required tangent plane since this plane contains two lines par-
allel to two lines of the required plane. Parallel to this plane and
tangent to the given cylinder the required plane may be drawn.
Let the drawing be made in accordance with these suggestions.

Construction. When the given surface is a cone. Through
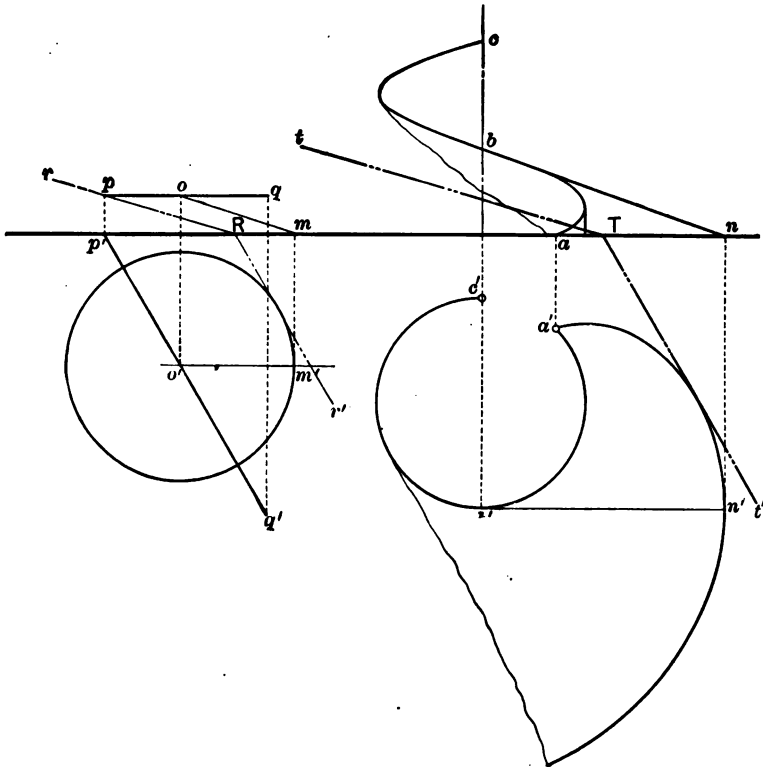
FIG. 145.

the apex of the cone draw a line parallel to the given line.
Through this line and tangent to the cone pass a plane. This
plane will be the required plane since it is tangent to the cone
and contains a line parallel to the given line. Let the drawing
be made in accordance with these suggestions.

Construction. Let the given surface be a convolute as in
Fig. 145. PQ is the given line to which the tangent plane is to

be parallel. Assume the point O on this line and through O draw OM parallel to BN, one of the elements of the convolute. Let the line OM generate a right cone whose base is a circle in V. Now tangent to the cone thus generated and through the line PQ pass the plane R. This plane will be parallel to the required plane since it contains the given line and also a line parallel to the element of contact on the convolute. Since the elements of the cone generated by OM make the same angle with V that the elements of the convolute do it is evident that the element of this cone along which rRr′ is tangent will be parallel to the element of contact of the convolute. Therefore, if tTt′ be drawn parallel to rRr′ and tangent to the convolute, it will be the required tangent plane.

150. Proposition 48. Given any single curved surface and its intersection with a plane to find the line tangent to this curve at a given point.

Discussion. Since the curve of intersection lies in a plane the tangent line will also lie in this plane. This required line will also lie in the plane which is tangent to the given surface at the given point. If, therefore, a plane be passed tangent to the given surface it will cut from the plane in which the given intersection lies a line which will be tangent to the given curve at the given point.

Construction. Let the given surface be a cone as in Fig. 146 with the given curve lying in plane sSs′. Let O be the point at which it is required to draw a tangent to the curve of intersection. Pass the plane rRr′ tangent to the cone at the point O (Proposition 45). Find the intersection of plane R with plane S (Proposition 8). This line is MN and it is tangent to the given curve at O.

151. This same method will apply to any problem wherein it is required to find the line tangent to any curve cut from a surface by a plane. Stated in general terms the method may be given as follows:

Pass a plane tangent to the surface at the given point. The intersection of this tangent plane with the plane cutting the given curve will always be tangent to this curve at the given point.

152. Proposition 49. Given any warped surface to find the plane tangent to it at a given point on the surface.

Discussion. The required tangent plane will contain the element of the surface through the point (the element of contact)
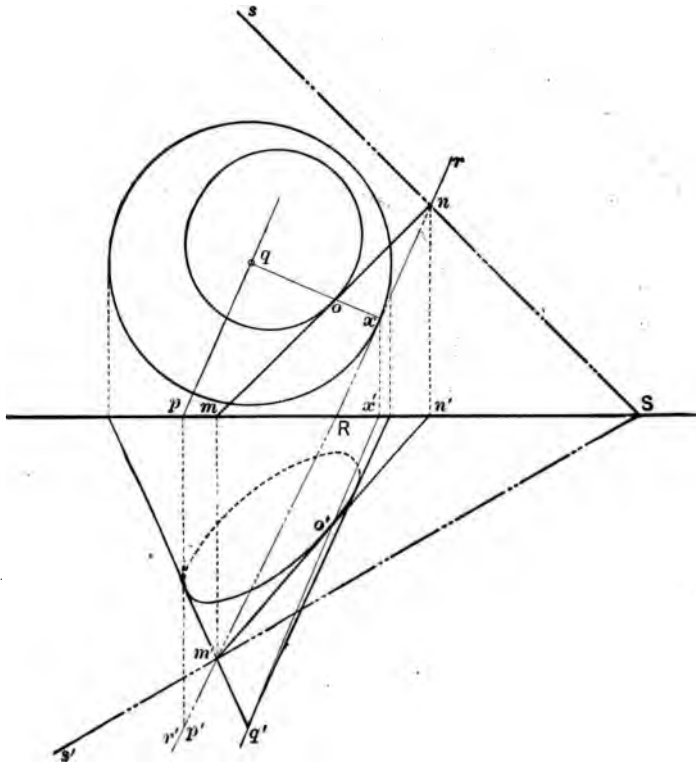
FIG. 146.

and the line tangent to any curve of the surface passing through the given point. These two lines will determine the required plane.

Construction. Let the given surface be an oblique helicoid as in Fig. 147 and let the point at which the tangent plane is to be drawn be A. Through A draw the element A2; this will be one line of the required plane. At A draw also AM tangent to the helix passing through A. (Art. 130.) This line will also lie

in the tangent plane.   Through AM and A2 pass the plane tTt′ which will be the required tangent plane.

It should be kept clearly in mind that a plane tangent to a warped surface at one point may cut the surface at a point very
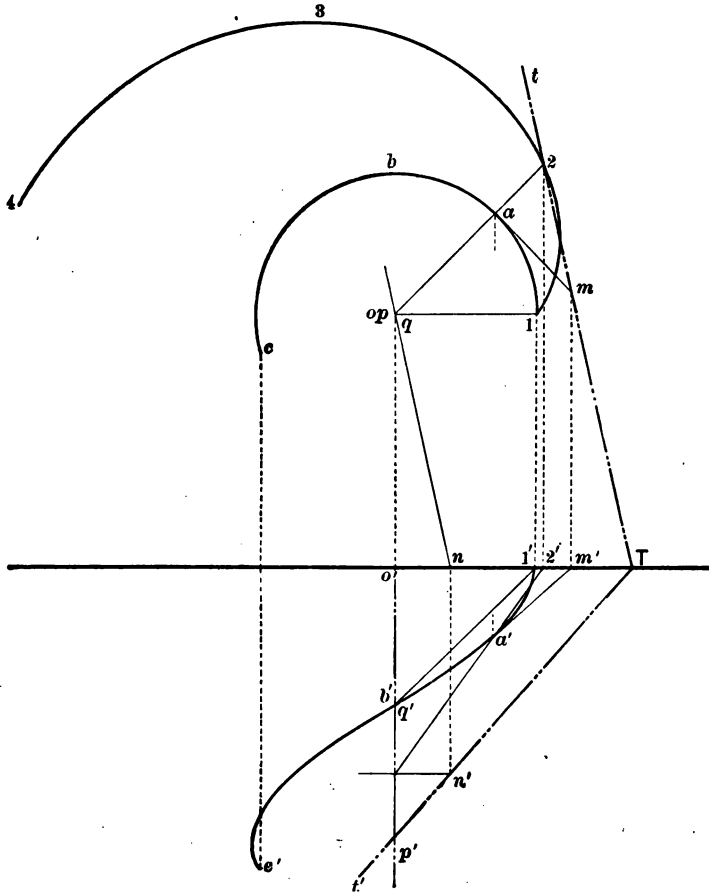
FIG. 147.

near this point.   And also that while the element of contact A2 lies in the required tangent plane this plane is *not tangent all along this element*.   From this it should be clear that in Fig. 147 tT is not tangent to the H base of the helicoid, and that the

tangent plane at any point will contain only lines tangent to curves of the surface through *that point.*

153. Proposition 50. Given any warped surface to find the plane tangent to it through any given point not on its surface.

Discussion. In general an indefinite number of planes may be passed through a point not on the surface tangent to a warped surface since an indefinite number of lines may be drawn through the given point tangent to curves of the surface. Any one of these tangent lines with the element which passes through the point of tangency will determine a plane which will contain the given point and be tangent to the given surface.

Construction. To find any one of these tangent planes pass a plane through the given point cutting a curve from the surface. Through the point draw a tangent to this curve. Through the point of contact of this tangent line draw an element of the given surface. The tangent line and this element will determine a plane which will meet the required conditions. Let the drawing be made in accordance with these suggestions,

154. Proposition 51. Given any warped surface to find the plane tangent to it which is parallel to a given line.

Discussion. If a plane be passed through the given surface parallel to the given line it will cut from the surface a curve to which it will be possible to draw a tangent which will be parallel to the given line. This tangent together with the element of the given surface through the point of contact of this tangent will determine a plane which will meet the required conditions. Since an indefinite number of planes may be passed through the surface parallel to the given lines there will be an equal number of possible tangent planes.

Construction. Let the construction be made in accordance with the given discussion.

155. Proposition 52. Given any double curved surface to find the plane tangent to it at a point on the surface.

Discussion. The plane tangent to a double curved surface will contain all the lines tangent to curves of the surface passing through the given point. If any two of these tangent lines be found they will determine the tangent plane at that point.

Construction. Let the given surface be an annular torus as in Fig. 148 and let the given point on its surface be O. Through O draw the line OQ tangent to the horizontal circle on the surface of the torus passing through O. Also draw the line OM tangent to that circle of the surface which lies in the meridian
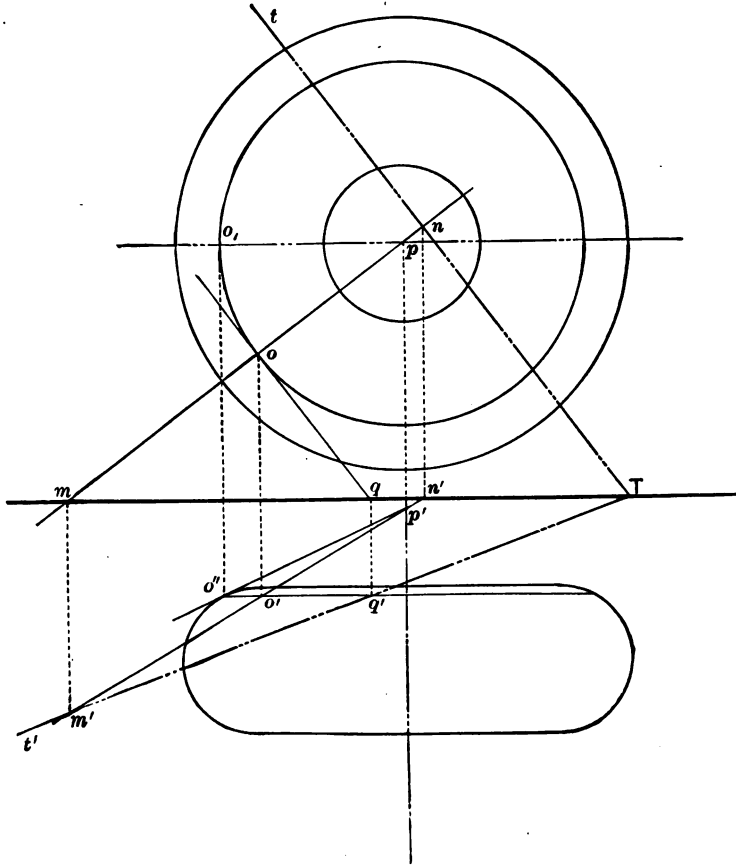
FIG. 148.

plane through O. (This tangent may be found by revolving O into the meridian plane parallel to V. In this position of O the tangent OP may be drawn and then revolved back to its true position with P, on the axis of revolution, remaining stationary.) These two tangent lines intersecting at the given

point O determine the required plane and the traces may be found by locating the piercing points of these lines, at M, Q, and N.

156. Proposition 53. Given any double curved surface to find the plane tangent to it through any given point not on the surface.

Discussion. If the given point be taken as the apex of a cone whose elements are all tangent to curves lying on the given surface it will be apparent that any plane tangent to this cone will also be tangent to the given surface. Such a plane, since it contains the given point and is tangent to the given surface, meets the required conditions. From this it may be seen that an indefinite number of planes may be passed through a point tangent to a surface.

157. Proposition 54. Given any double curved surface to find the plane tangent to it and parallel to any given line.

Discussion. If a series of lines be drawn tangent to curves lying on the given surface and parallel to the given line these lines will be the elements of a cylindrical surface since they are all parallel. Any plane then tangent to this cylindrical surface will also be tangent to the given surface and parallel to the given line. Since there are an infinite number of such planes there are an indefinite number of planes which will meet the required conditions.

158. Proposition 55. Given any two surfaces and their curve of intersection to find a line tangent to this curve at a given point.

Discussion. Since the line is tangent to a curve of each surface it must lie in a plane tangent to the surface at the given point. If, therefore, planes be passed tangent to the given surfaces at the given point and their line of intersection found this line will be tangent to the curve of intersection of the two surfaces at the given point.

Construction. Let the given surfaces be two cylinders as shown in Fig. 149 and let O be the point on their curve of intersection at which a tangent is to be drawn. Through O pass a plane tangent to each cylinder (Proposition 45). Plane sSs' is
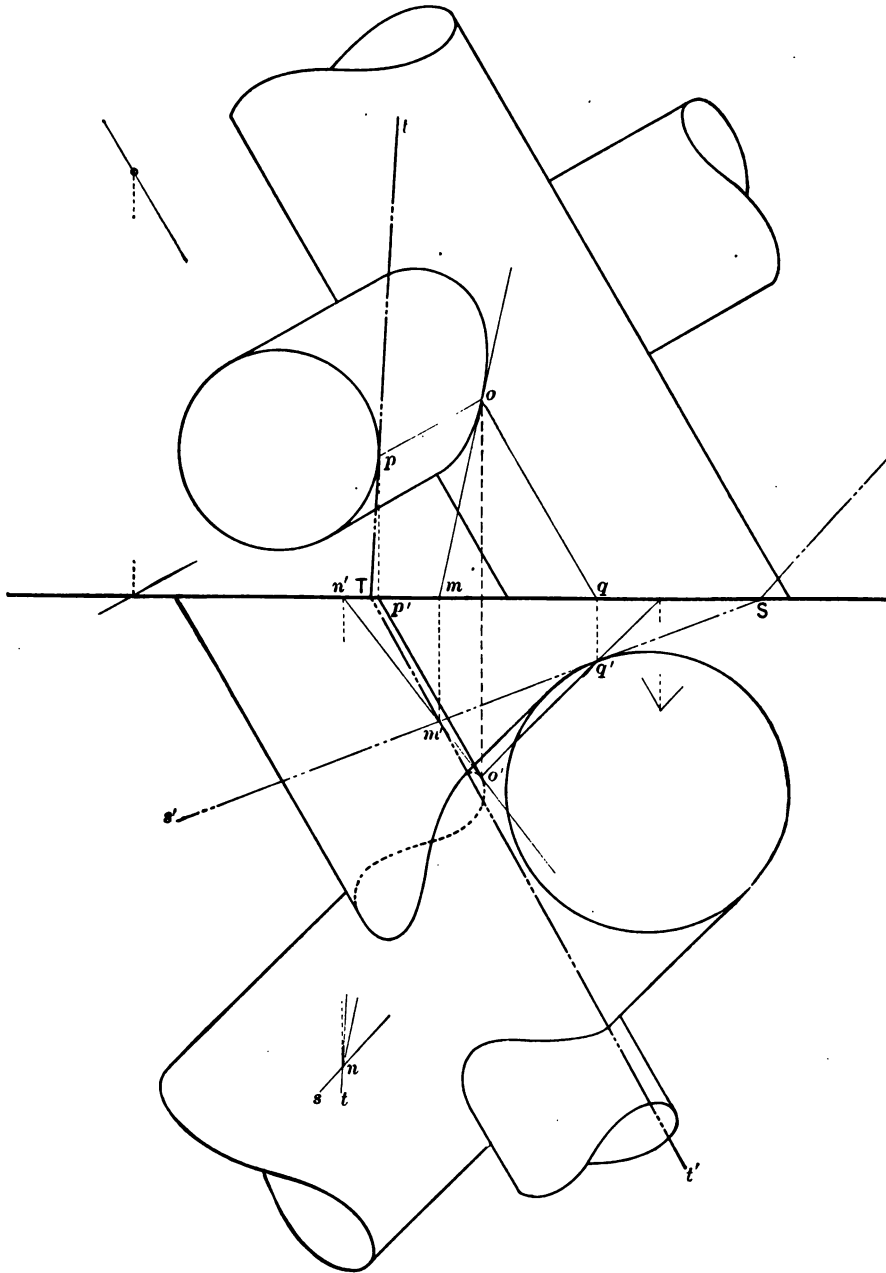
FIG. 149.

tangent to the right hand cylinder and plane tTt' is tangent to the left hand cylinder. Find the intersection of these two tangent planes (Proposition 8). One pair of traces intersect at N and the other pair at M thus giving MN as the line of intersection. When extended MN passes through the given point O and the line MO is tangent to the curve of intersection at the given point.

PROBLEMS ON TANGENT PLANES AND TANGENT LINES

275. Assume a cylinder with a circular base in H and with its axis oblique to H and piercing V. Pass a plane tangent to this cylinder at any assumed point. Is the V trace of the tangent plane tangent to the V base of the cylinder?

276. Assume a cone whose base is a circle in V and whose axis is oblique to H and V. Pass a plane tangent to this cone at any assumed point on its surface.

277. Assume a cylinder with its base in 4 and its axis oblique to H and V. Through any point assumed outside the surface of the cylinder pass a plane tangent to the surface. Give both solutions.

278. Assume a right circular cylinder with its axis parallel to the GL. Pass a plane tangent to this cylinder through a point assumed outside its surface. Give both solutions.

279. Assume a right circular cone with its base in H. Assume a line MN. Find the plane tangent to the cone which will be parallel to MN. (There are some positions of the line MN which would make the problem impossible. What are they?)

280. At some point on the curve of intersection in Problem 193 draw a line tangent to the curve.

281. Assume a point on the curve of intersection in Problem 215 and at this point draw a line tangent to the curve.

282. The axis of an oblique helicoid is perpendicular to H 3" in front of V. The generating line inclines 60 degrees to H and pierces H 2" from the foot of the axis in its initial position. The pitch is 3". Pass a plane tangent to the helicoid at a point which the initial piercing point of the generatrix would assume after one-eighth of a turn.

283. Pass a plane tangent to an assumed point on the surface of the conoid in Problem 258.

284. Given a sphere to pass a plane tangent to it at a given point on its surface. Work this problem by an original method. (Hint — a plane tangent to a sphere will be perpendicular to the radius at the point of contact.)

285. Assume an ellipsoid of revolution and a point on its surface. Pass a plane tangent to the ellipsoid at the given point.

286. Assume an hyperboloid of revolution similar to the one shown in Fig. 138 and pass a plane tangent to it at any point on its surface.

287. Assume a point on the curve of intersection of any of the single curved surface problems in Chapter XV and pass a line tangent to the curve at this point.

288. Assume a point of the curve of intersection of any of the problems in Chapter XVI and find the line tangent to the curve at this point.

CHAPTER XIX

MODEL MAKING

159. One of the most interesting and practical means of studying the development of surfaces and the laying out of patterns for them consists in the construction of actual scale models. This not only affords practice in the use of drawings and gives experience in constructing the object represented in the drawing, but it is also an excellent means of showing how curves of intersection appear when laid on a pattern as well as
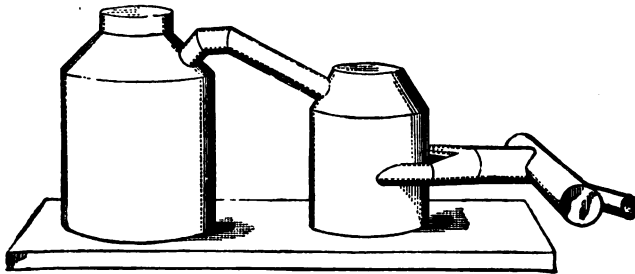
FIG. 150. — A paper model of a blast furnace piping problem constructed to scale from ordinary detail paper and mounted on a wooden base.

when they are an actual part of the object on which they lie. While such model making must of course be done to scale yet the experience acquired serves to illustrate many of the practical considerations which must be kept in mind when laying out sheet metal work.

160. Practical Hints. In making paper models such as the one shown in Fig. 150 a good grade of detail paper will give satisfactory results. After the developments of the several surfaces have been accurately made they may be cut out — leaving an allowance along the proper edges of the pattern for lap — and glued together, thus forming the actual surface.
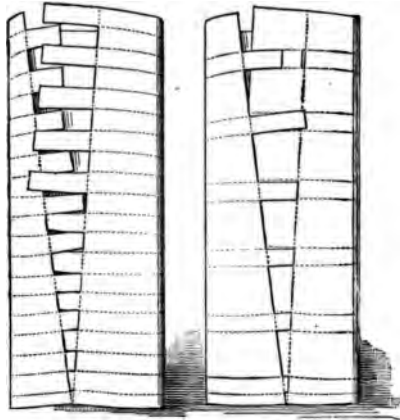
FIG. 151. — Illustration showing two methods of making joints in paper model making. The method at the right gives better results for joints along straight lines while the one at the left is better for curved joints.
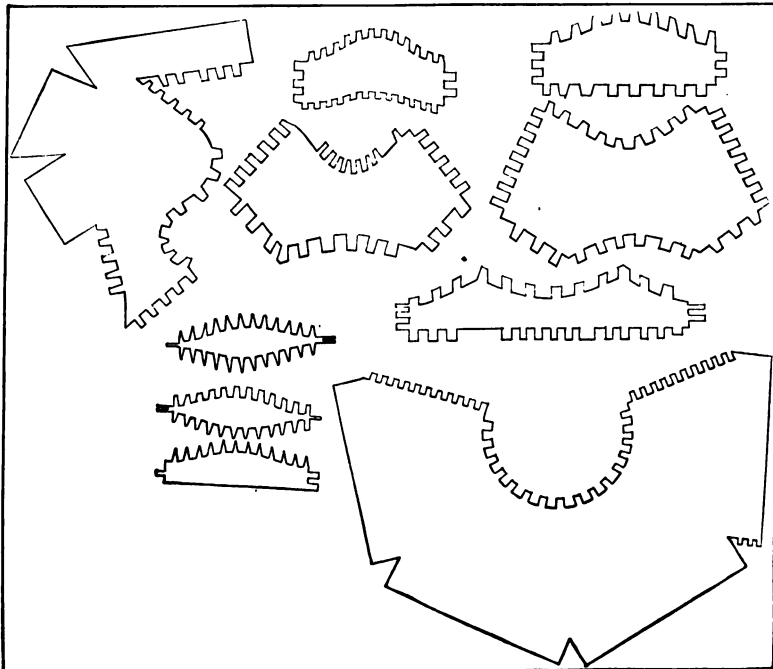
FIG. 152. — Patterns for the parts of the model shown in Fig. 155 before being glued together   Note the method for making the joints.

Two convenient methods for making the joint along an element of the surface are shown in Fig. 151. The joint in the cylinder on the right is overlapped and on the one side of the pattern tongues are cut to fit into the slits on the other side of the pattern. This method has been found to give good results and is somewhat easier than the one shown on the cylinder at the left in Fig. 151. For joints of two intersecting surfaces it will be found more convenient to cut tongues on both patterns, the tongues on one side being arranged to fit into spaces between the tongues on the other as shown in the cylinder at the left in Fig. 151. It requires a little care and foresight to get the tongues and spaces to knit but the resulting joint is so much easier to form and is so much more secure that the extra trouble is worth the time it takes.

Both library paste and glue are good " solder " for · t h e joints. Library paste is pleasanter to work with and takes less time to dry but it seems to hold less well than glue. A glued joint has been found to last indefinitely but joints fastened with paste are apt to break open in time.

FIG. 153. — A view of the model shown in Fig. 154 cut open to show the method of securing stiffness and true form by inserting cardboard forms.

161. The tendency of surfaces such as cones and cylinders when formed from detail paper patterns is to be flat along the joints. This is due to the extra stiffness at these points caused by overlapping edges. An easy method of overcoming this and securing rigidity in the model is to insert cardboard reinforcements as shown in Fig. 153. These cardboard reinforcements may be made in the form of right sections and glued in place. The model will thus be made to keep its proper shape and will be much stiffer.

162. Naturally a model such as the one shown in Fig. 154 will get more or less soiled from handling during its construction and

its appearance may be greatly improved if upon completion it is given a coat of bronze or silver paint. This covers the glue marks and gives the model a fresh and finished appearance at a trifling cost.
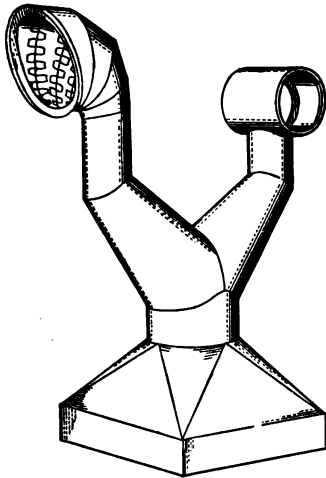
FIG. 154. — A paper model. A close examination of the opening in the cowl will show the method used to make the joints.
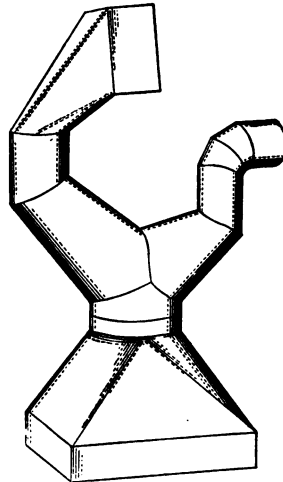
FIG. 155. — Another paper model.
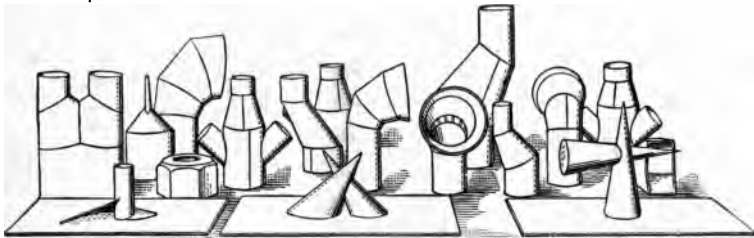
FIG. 156. — A collection of paper models made by the methods just described. With the exception of two or three all of these models were made by students during one course.

The models which are used here for illustration, both the finished models and the ones which have been cut open to show methods of construction, were made by students in the regular required course in descriptive geometry at the University of

Iowa. This fact is mentioned to impress upon students who use this text that work of this character and quality is quite within their capacity.

GENERAL PROBLEMS

289-302. Draw a plan and elevation of the object shown in the drawing and lay out patterns for each part. After the patterns have been laid out to the same scale as the drawing, they are to be fitted together and glued to form a model of the object.

PROBLEM 289. HOPPER

	A	B	C	D	E	F	G
a	12″	12″	2″	5″	6″	5″	2″
b	36″	18″	6″	0″	18″	0″	6″
c	36″	18″	6″	0″	18″	15″	6″
d	36″	18″	6″	6″	18″	0″	6″
e	30″	12″	4″	4″	12″	9″	12″

PROBLEM 290. RECTANGULAR TAPERED CONNECTING PIPE

	A	B	C	D	E	F	G	H	L	K
a	12″	6″	6″	12″	6″	6″	6″	24″	6″	3″
b	12″	3″	6″	9″	8″	6″	5″	36″	6″	4½″
c	12″	4″	6″	9″	8″	6″	5″	36″	6″	4″
d	12″	6″	6″	12″	6″	6″	6″	24″	6″	6″
e	10″	10″	6″	6″	3″	3″	6″	20″	6″	0″

PROBLEM 291. TWISTED CONNECTING PIPE

	A	B	C	D	E	F	G
a	12″	6″	12″	6″	3″	3″	22″
b	18″	12″	20″	9″	6″	6″	30″
c	8″	8″	12″	6″	4″	4″	24″
d	16″	8″	16″	8″	6″	6″	30″
e	8″	4″	8″	4″	6″	6″	24″

PROBLEM 292. CONICAL CONNECTION FOR OFFSET CYLINDERS

	A	B	C	D	E	X	Notes
a	8″	16″	8″	8″	14″	60	
b	12″	18″	12″	10″	12″	45	
c	8″	16″	8″	8″	14″	30	
d	12″	18″	12″	10″	12″	30	
e	8″	14″	10″	8″	18″	60	

PROBLEM 293. HOPPER FOR PIPE

	A	B	C	D	E	F	G	X	Pipe Section
a	12″	6″	6″	10″	3″	5″	10″	90	Rect.
b	15″	15″	10″	18″	5″	9″	12″	90	Circle
c	6″	6″	6″	10″	3″	5″	10″	90	Circle
d	15″	15″	10″	18″	5″	9″	12″	120	Circle
e	9″	9″	6″	10″	3″	5″	10″	75	Circle

Nᴏᴛᴇ. In all of the above problems the axis of the hopper remains vertical.

PROBLEM 294. CONICAL HOPPER ON CYLINDRICAL PIPE

	A	B	C	D	E	X	Y
a	12″	12″	14″	12″	12″	90	90
b	12″	12″	12″	8″	8″	90	90
c	18″	12″	20″	12″	18″	75	90
d	12″	12″	12″	8″	8″	90	75
e	12″	12″	20″	12″	18″	90	90

PROBLEM 295. INTERSECTION OF CYLINDRICAL PIPES

	A	B	C	D	E	X	Y
a	12″	12″	16″	6″	6″	45	90
b	18″	18″	20″	10″	10″	60	90
c	10″	10″	20″	6″	6″	45	90
d	12″	12″	20″	9″	9″	60	90
e	18″	18″	20″	10″	10″	105	90

PROBLEM 296.   REDUCING BREECHING

	A	B	C	D	E	F	G	H	X	Y
a	6″	6″	14″	8″	8″	8″	6″	6″	45	45
b	6″	8″	18″	12″	18″	10″	6″	10″	60	45
c	8″	6″	16″	8″	8″	8″	6″	4″	60	45
d	8″	8″	18″	12″	12″	12″	8″	8″	30	30
e	6″	6″	16″	10″	10″	10″	8″	8″	30	60

PROBLEM 297. CONNECTING ROD END

	A	B	C	D	E	D′	R
a	3″	6″	12″	1½″	8″	2½″	1½″
b	4″	8″	12″	4″	10″	5″	3″
c	3″	6″	12″	2½″	8″	3½″	2″
d	4″	8″	10″	2″	10″	3″	4″
e	3″	6″	8″	1½″	6″	3″	2″

NOTE: Make model of rectangular part only.

PROBLEM 298.   COWL FOR SHIP'S VENTILATOR

	A	B	C	D	E	X	Number of Sections
a	12″	6″	18″	8″	14″	105	Five
b	18″	9″	24″	9″	18″	105	Five
c	18″	9″	24″	9″	14″	105	Five
d	12″	6″	18″	· 8″	14″	90	Two
e	14″	7″	20″	9″.	12″	105	Five

PROBLEM 299. TRANSITION PIECE

	A	B	C	D	E	F	G	H
a	12″	4″	8″	4″	6″	12″	6″	6″
b	12″	4″	8″	4″	6″	12″	3″	6″
c	12″	4″	8″	4″	6″	8″	4″	3″
d	12″	4″	8″	4″	6″	12″	3″	3″
e	12″	4″	8″	4″	4″	12″	6″	10″

PROBLEM 300.   VENTILATOR COWL WITH Y-CONNECTION AND SQUARE HOOD

	A	B	C	D	E	F	G	H	K	L	M	N	O	X	Y	Z
a	32″	6″	12″	26″	18″	16″	12″	18″	8″	4″	18″	4″	8″	45	45	105
b	32″	6″	12″	26″	18″	16″	12″	18″	8″	4″	18″	4″	8″	60	45	105
c	36″	6″	12″	30″	14″	18″	10″	12″	12″	6″	18″	6″	8″	60	60	105

PROBLEM 301. SQUARE HOOD WITH Y-CONNECTION, ELBOW, AND
TRANSITION PIECE

	A	B	C	D	E	F	G	H	K	L	M	N	O	P	X	Y	Z
a	32″	6″	12″	26″	18″	16″	8″	18″	12″	8″	8″	18″	8″	8″	45	45	60
b	32″	6″	12″	26″	18″	16″	8″	20″	10″	8″	8″	18″	8″	6″	60	45	45
c	32″	6″	12″	26″	18″	16″	8″	20″	10″	8″	8″	18″	8″	8″	75	60	45

PROBLEM 302. DUST CATCHER AND GAS MAIN

CHAPTER XX

APPENDIX

The following geometrical constructions will be found useful in drawing many of the problems in this text. While it is understood that many of them are well known problems in plane geometry it has been thought well to include them here for the convenience of the student.

It is to be understood that no attempt has been made to prove any of the constructions, and also that the particular construction given may not be the only method of arriving at the same result.

The attempt has been made to draw the figure in such a way that the construction will be more or less obvious, and therefore the explanation has been made as brief as possible. The methods here given have been found by experience to be the most practical constructions for this kind of work.
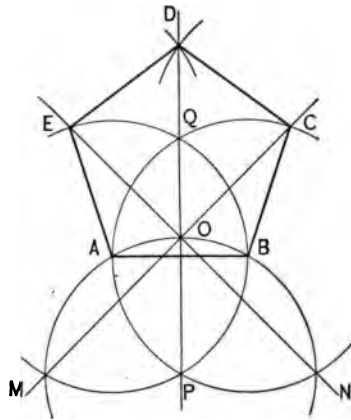
Given the length of one side to construct a regular pentagon.

Let AB be the given side. With AB as a radius and B as a center draw the arc NPAQC. With A as a center and AB as the radius draw the arc MPBQE. With P as a center and the same radius draw the arc MAOBN. Draw the lines PQ, MC, NE, thus locating E and C. With E and C as centers and AB as radius draw arcs intersecting at D. ABDCE, then, is the required regular pentagon.

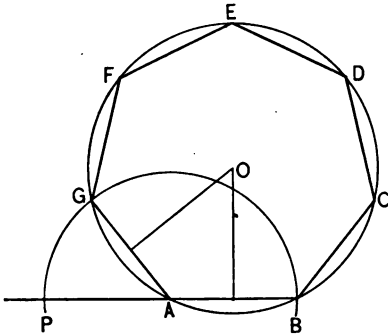**Given the length of one side to construct a regular hexagon.**

Let AB be the given side. With A and B as centers and AB as radii draw arcs intersecting at O. This will be the center of the hexagon. With O as a center and a radius equal to OA draw the circumscribing circle. Lay off chords on this circle equal in length to AB, thus locating points C, D, E, and F. ABCDEF is the required hexagon.
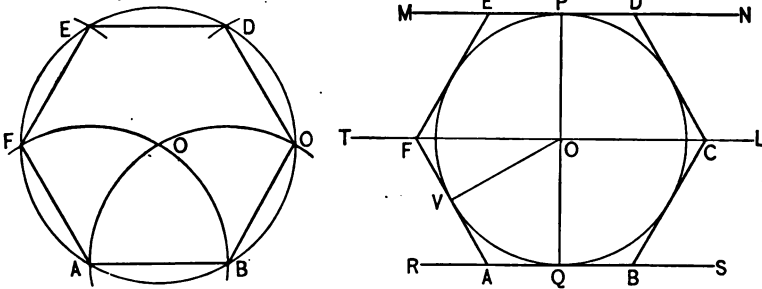
Given the distance "across the flats" to construct a regular hexagon.

Let MN and RS be two lines representing the sides of a hexagon. Draw PQ perpendicular to MN and RS, and bisect it by the perpendicular TL. The point O is the center of the inscribed circle of the hexagon. Draw OV making an angle of 60 degrees with OQ. Draw also the inscribed circle VQP with O as a center and OQ as a radius. At V draw AF tangent to OV; AF is the length of one side of the required hexagon which may now be constructed by any convenient method.

**Given the length of one side to construct a regular polygon of any given number of sides.**

Let AB be the given side and let it be required to draw a polygon with seven sides. With A as a center and a radius equal to AB draw the arc BOGP. Divide this arc into seven equal parts

and through the second point, G, draw the line AG. This will be the side of the polygon adjacent to AB. Bisect AB and AG by perpendiculars meeting at O. This point O will be the center of the circumscribing circle. Draw this circle and lay off on it seven chords each equal to AB, thus locating the points C, D, E, and F. ABCDEFG, then, is the required polygon of seven sides.
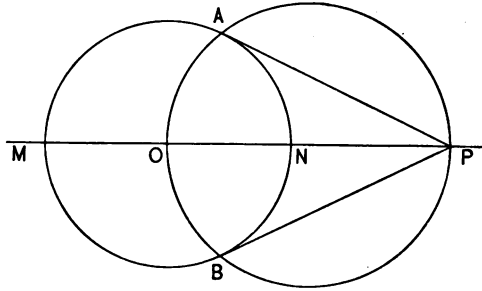
To draw a tangent to a circle from a given point without the circumference.

Let MANB be the given circle whose center is at O, and let P be the given point from which the tangent is to be drawn. Draw OP and upon OP as a diameter describe the circle OABP cutting the given circle at the points A and B. Draw PA and PB; these will be the required tangents from P and A and B will be the tangent points.
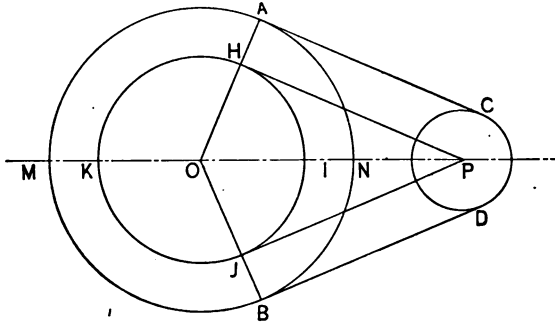

To draw a line tangent to two given circles.

Let MANB be one circle and CD the other circle to which the line is to be drawn tangent. Draw PO connecting the centers. Lay off NI equal to PD and describe the circle HIJK. This circle is equal in radius to the difference between the radii of the two given circles.

Now by the previous problem draw the tangents PJ and PH

from P to the circle HIJK.   Extend OH to cut the given circle
at A and extend OJ to cut at B.   A and B will be the points of

tangency and AC and BD drawn from these points parallel to HP and JB will be the two lines which may be drawn tangent to the two given circles.
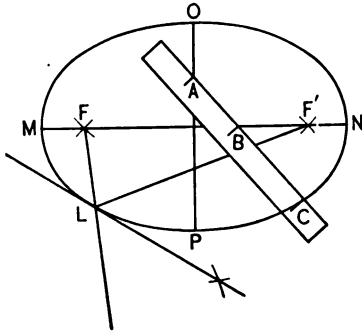
To draw an ellipse by trammels, given the axes.

Let MN be the major axis and OP the minor axis of the ellipse. Cut a strip of paper as shown and lay off AC equal to half of MN and lay off BC equal to half of OP.   Now keeping the point A always on the axis OP and the point B always on the axis MN move the trammel about and mark the various positions of C. A sufficient number of points may be thus marked to enable the curve to be accurately drawn in with the irregular curve as a guide.

**To locate the foci of the ellipse.**

With P as a center and half of MN as a radius draw arcs cutting MN at F and F', the required foci.

To draw a tangent to an ellipse at a point on the curve.

Let L be the given point. Draw LF and LF', the focal radii.   Bisect the exterior angle between LF and LF' and draw the bisector RL, the required tangent.

**To draw an ellipse, given the axes.   Second Method.**

Draw the circle upon MN, the major axis; and draw also the circle upon PQ, the minor axis.   Draw o1, o2, o3, etc.   From a,
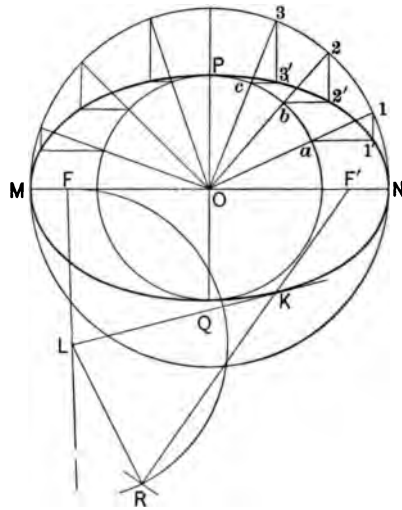
b, c, etc., draw horizontal lines; from 1, 2, 3, etc., draw vertical lines. Locate the intersections of these lines as at 1', 2', 3', etc., and find a sufficient number so that a curve may be drawn through them as shown. This curve will be the required ellipse.

To draw a tangent to an ellipse from a point without the curve.

Locate F and F', the foci of the given ellipse. Let L be the given point at which the tangent is to be drawn. Draw LF, and with LF as a radius and L as a center describe the arc FR. With F' as a center and MN as a radius describe an arc cutting the arc FR at R, and draw RF'. The point where this line RF' cuts the ellipse or K is the point of tangency and LK, then, is the required tangent.
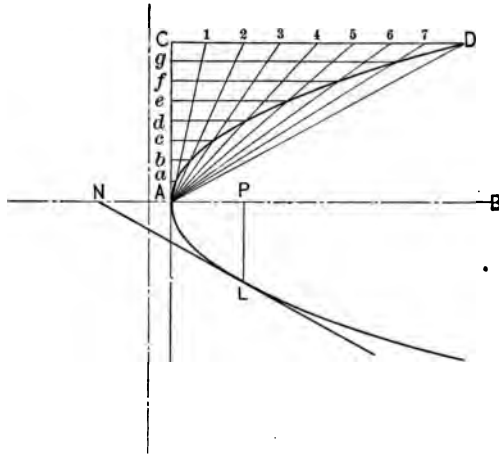
To draw a parabola when its axis, its vertex, and a point on the curve are given.
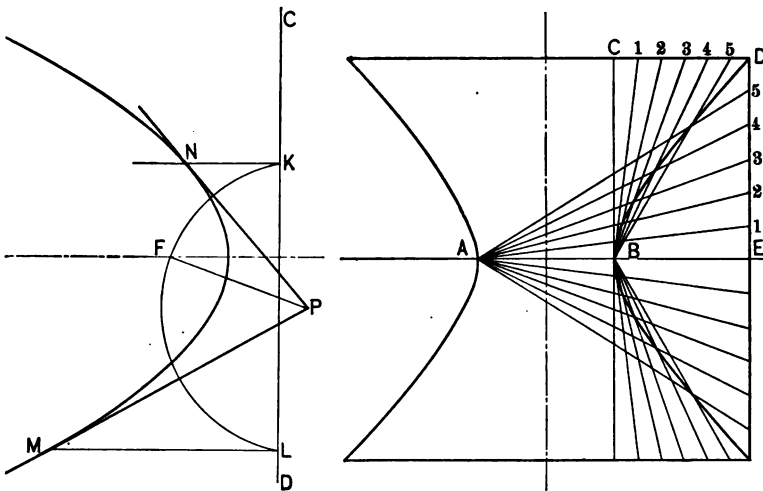
Let AB be the axis, A the vertex, and D a point on the curve. Draw DC and AC and divide each into the same number of equal parts, thus locating points 1, 2, etc., and a, b, c, etc. Draw A1, A2, A3, A4, A5, A6, A7, and find where each of these lines is intersected by lines through a, b, c, d, e, f, g drawn parallel to AB. The intersections thus located will be points on the required parabola, and when a sufficient number of them have been found the curve may be drawn with the irregular curve.

To draw a tangent to a parabola at a point on the curve.

Let L be the given point.   Draw LP perpendicular to AB and make NA equal to PA.   Draw NL; this will be the required tangent to the curve at L.

To draw a tangent to a parabola from a point not on the curve.

Let CD be the directrix of the given parabola whose focus is F, and let P be the point from which the line is to be drawn tan-
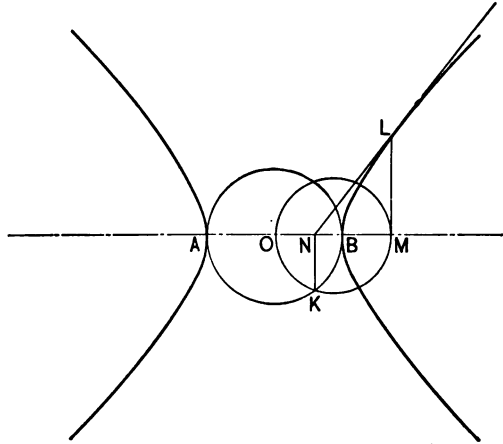
gent to the curve.   With P as a center and a radius Pf describe the arc cutting the directrix at K and L.   From these points draw lines LM and KN perpendicular to CD cutting the parabola at M and N.   M and N, then, are the tangent points, and lines drawn from P to these points will be tangent to the curve.

To draw an hyperbola, given the major axis and one point on the curve.

Let AB be the given axis and D the given point.   Draw the rectangle BCDE, and divide CD and ED into any number of equal parts, in this case six.   Draw A5, A4, A3, A2, A1, also B5, B4, B3, B2, B1.   At the intersection of each of these lines from A with the corresponding lines from B will be located a point on the curve, and by finding a sufficient number of points the curve may be drawn as shown.

**To draw a tangent to an hyperbola at a given point on the curve.**

Let AB be the major axis of the hyperbola and L the given point at which the tangent is to be drawn. Draw LM perpen-

dicular to AB. On OM as a diameter describe the circle cutting the major auxiliary circle at K. Draw KN perpendicular to AB. Draw NL, the required tangent.

To rectify a given arc. An approximate method.

Let AB be the given arc. Divide it as shown into a number of equal parts. Let the distances into which the arc is divided, as A1, 12, 23, 3B, be made small enough so that the chord sub-tending this small arc is practically equal to the arc itself. With the bow dividers lay off on the given straight line these distances. The distance AB on the straight line, then, will be equal to the distance AB on the arc.
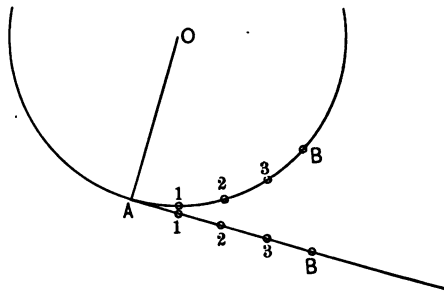
If it be desired to lay off on a given arc any given length the same method may be used. The whole point of this method

is in keeping the distances so small that there is no difference, practically, between the arc and its subtending chord.

Other methods of laying off arcs onto straight lines may be used, but the above is by far the most convenient for the . draftsman.

To measure an angle by its natural tangent.

Let the given angle to be measured be ACB. Its natural tangent is $\dfrac{AB}{BC}$. Divide, therefore, AB by BC and look up the value of the angle corresponding to this quotient in any reliable table of natural tangents. The result will be the value of the angle in degrees and minutes.
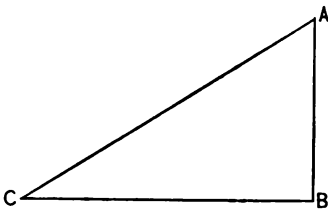
To lay off a given angle by its natural tangent.

Lay off B any convenient distance from C, say 3″, and erect a perpendicular BA. To find the length of BA look up the value of the natural tangent of the given angle in any reliable table of natural tangents, multiply it by three, and lay off this distance on BA, thus locating A. Connect A with C and BCA then will be the required angle.

NOTE. Since tables of natural tangents are computed on a base of 1 the value given for the natural tangent must always be multiplied by the length of the base when it is taken greater than unity.